Bootstrapping holographic defect correlators in $\mathcal{N} = 4$ SYM

Aleix Gimenez-Grau

DESY Hamburg

Based on: Barrat, AGG, Liendo: 2108.13432.

The main characters

Stress-tensor (20') multiplet

$$\mathcal{O}_2 = \mathrm{tr}\, \phi^{\{i_1} \phi^{i_2\}}$$

$$\langle \mathcal{O}_2 \ldots \mathcal{O}_2 \rangle$$

Wilson loop

$$\mathcal{W} = \frac{1}{N} \mathrm{tr}\, P e^{\int d\tau (i A_\tau + \phi_6)}$$

$$\langle \mathcal{W}_{\textsf{circle}} \rangle = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$$

The main characters

Stress-tensor (20') multiplet

$$\mathcal{O}_2 = \operatorname{tr} \phi^{\{i_1} \phi^{i_2\}}$$

$$\langle \mathcal{O}_2 \dots \mathcal{O}_2 \rangle$$

Wilson loop

$$\mathcal{W} = \frac{1}{N} \operatorname{tr} P e^{\int d\tau (i A_\tau + \phi_6)}$$

$$\langle \mathcal{W}_{\textsf{circle}} \rangle = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$$

We study their mixed correlators!

The main characters

Stress-tensor (20') multiplet

$$\mathcal{O}_2 = \operatorname{tr} \phi^{\{i_1} \phi^{i_2\}}$$

$$\langle \mathcal{O}_2 \dots \mathcal{O}_2 \rangle$$

Wilson loop

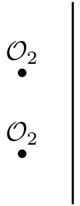
$$\mathcal{W} = \frac{1}{N} \operatorname{tr} P e^{\int d\tau (i A_\tau + \phi_6)}$$

$$\langle \mathcal{W}_{\text{circle}} \rangle = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$$

We study their mixed correlators!

The simplest non-trivial observable is

$$\langle\!\langle\, \mathcal{O}_2 \mathcal{O}_2 \,\rangle\!\rangle \equiv \frac{\langle \mathcal{W} \mathcal{O}_2 \mathcal{O}_2 \rangle}{\langle \mathcal{W} \rangle}$$



Strong coupling expansion

Our regime: planar $\mathcal{N} = 4$ SYM at large $\lambda = g^2 N$.

Strong coupling expansion

Our regime: planar $\mathcal{N} = 4$ SYM at large $\lambda = g^2 N$.

The expansion takes the form: [Giombi, Pestun '12]

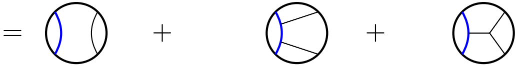
$$\langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle = \langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle^{(0)} + \frac{\lambda}{N^2}\left(\langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle^{(1)} + \frac{1}{\sqrt{\lambda}}\langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle^{(2)} + \dots\right) + \dots$$

Strong coupling expansion

Our regime: planar $\mathcal{N} = 4$ SYM at large $\lambda = g^2 N$.

The expansion takes the form: [Giombi, Pestun '12]

$$\langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle = \langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle^{(0)} + \frac{\lambda}{N^2}\left(\langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle^{(1)} + \frac{1}{\sqrt{\lambda}} \langle\!\langle\, \mathcal{O}\mathcal{O} \,\rangle\!\rangle^{(2)} + \ldots \right) + \ldots$$

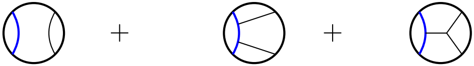


Strong coupling expansion

Our regime: planar $\mathcal{N} = 4$ SYM at large $\lambda = g^2 N$.

The expansion takes the form: [Giombi, Pestun '12]

$$\langle\!\langle \mathcal{O}\mathcal{O} \rangle\!\rangle = \langle\!\langle \mathcal{O}\mathcal{O} \rangle\!\rangle^{(0)} + \frac{\lambda}{N^2}\left(\langle\!\langle \mathcal{O}\mathcal{O} \rangle\!\rangle^{(1)} + \frac{1}{\sqrt{\lambda}} \langle\!\langle \mathcal{O}\mathcal{O} \rangle\!\rangle^{(2)} + \dots \right) + \dots$$

$=$ 

In this work:

- Focus on the leading non-trivial correction $\langle\!\langle \mathcal{O}\mathcal{O} \rangle\!\rangle^{(2)}$
- We **do not** calculate the Witten diagram
- We **bootstrap** the result with an inversion formula

Crossing equation

Our two-point function is

$$\langle\!\langle \mathcal{O}_2 \mathcal{O}_2 \rangle\!\rangle \;\propto\; \mathcal{F}(\underbrace{z, \bar{z}}_{\substack{\text{spacetime} \\ \text{cross-ratios}}}, \sigma) \;=\; \underbrace{\sigma^2 F_0 + \sigma F_1 + F_2}_{R-\text{symmetry channels}}$$

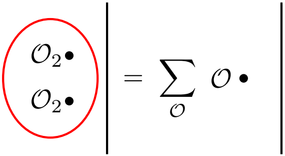
Crossing equation

Our two-point function is

$$\langle\!\langle \mathcal{O}_2 \mathcal{O}_2 \rangle\!\rangle \; \propto \; \mathcal{F}(\underbrace{z, \bar{z}}_{\text{spacetime cross-ratios}}, \sigma) \; = \; \underbrace{\sigma^2 F_0 + \sigma F_1 + F_2}_{R-\text{symmetry channels}}$$

Bulk-bulk : $\quad \mathcal{O}_2 \times \mathcal{O}_2 \sim \sum_{\mathcal{O}} \lambda_{22\mathcal{O}} \mathcal{O}$

Bulk-defect : $\quad \mathcal{O}_2 \sim \sum_{\widehat{\mathcal{O}}} b_{2\widehat{\mathcal{O}}} \widehat{\mathcal{O}}$



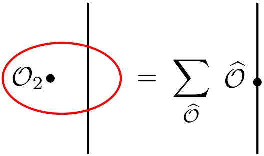
Crossing equation

Our two-point function is

$$\langle\!\langle\, \mathcal{O}_2 \mathcal{O}_2 \,\rangle\!\rangle \;\propto\; \mathcal{F}(\underbrace{z, \bar{z}}_{\text{spacetime cross-ratios}}, \sigma) \;=\; \underbrace{\sigma^2 F_0 + \sigma F_1 + F_2}_{R-\text{symmetry channels}}$$

Bulk-bulk : $$\mathcal{O}_2 \times \mathcal{O}_2 \sim \sum_{\mathcal{O}} \lambda_{22\mathcal{O}} \mathcal{O}$$

Bulk-defect : $$\mathcal{O}_2 \sim \sum_{\widehat{\mathcal{O}}} b_{2\widehat{\mathcal{O}}} \widehat{\mathcal{O}}$$



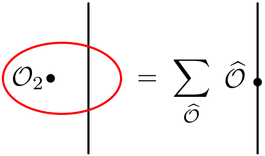
Crossing equation

Our two-point function is

$$\langle\!\langle\, \mathcal{O}_2 \mathcal{O}_2 \,\rangle\!\rangle \;\propto\; \mathcal{F}(\underbrace{z, \bar{z}}_{\text{spacetime cross-ratios}}, \sigma) \;=\; \underbrace{\sigma^2 F_0 + \sigma F_1 + F_2}_{R-\text{symmetry channels}}$$

Bulk-bulk : $\mathcal{O}_2 \times \mathcal{O}_2 \sim \sum_{\mathcal{O}} \lambda_{22\mathcal{O}} \mathcal{O}$

Bulk-defect : $\mathcal{O}_2 \sim \sum_{\widehat{\mathcal{O}}} b_{2\widehat{\mathcal{O}}} \widehat{\mathcal{O}}$



Our correlator $\langle\!\langle\, \mathcal{O}_2 \mathcal{O}_2 \,\rangle\!\rangle$ contains an infinite amount of CFT data

$$\mathcal{F}(z, \bar{z}, \sigma) = \sum_{\mathcal{O}} \lambda_{22\mathcal{O}} a_{\mathcal{O}} \, \mathcal{G}_{\mathcal{O}}(z, \bar{z}, \sigma) = \sum_{\widehat{\mathcal{O}}} b^2_{2\widehat{\mathcal{O}}} \, \hat{\mathcal{G}}_{\widehat{\mathcal{O}}}(z, \bar{z}, \sigma)$$

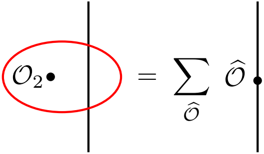
Crossing equation

Our two-point function is

$$\langle\!\langle\, \mathcal{O}_2 \mathcal{O}_2 \,\rangle\!\rangle \;\propto\; \mathcal{F}(\underbrace{z, \bar{z}}_{\text{spacetime cross-ratios}}, \sigma) \;=\; \underbrace{\sigma^2 F_0 + \sigma F_1 + F_2}_{R-\text{symmetry channels}}$$

Bulk-bulk : $\mathcal{O}_2 \times \mathcal{O}_2 \sim \sum_{\mathcal{O}} \lambda_{22\mathcal{O}} \mathcal{O}$

Bulk-defect : $\mathcal{O}_2 \sim \sum_{\widehat{\mathcal{O}}} b_{2\widehat{\mathcal{O}}} \widehat{\mathcal{O}}$



Our correlator $\langle\!\langle\, \mathcal{O}_2 \mathcal{O}_2 \,\rangle\!\rangle$ contains an infinite amount of CFT data

$$\mathcal{F}(z, \bar{z}, \sigma) = \sum_{\mathcal{O}} \lambda_{22\mathcal{O}} a_{\mathcal{O}}\, \mathcal{G}_{\mathcal{O}}(z, \bar{z}, \sigma) = \sum_{\widehat{\mathcal{O}}} b^2_{2\widehat{\mathcal{O}}}\, \hat{\mathcal{G}}_{\widehat{\mathcal{O}}}(z, \bar{z}, \sigma)$$

Reverse logic: consistency with crossing fixes $\mathcal{F}$!

Inversion formula

The central tool is the inversion formula: [Caron-Huot; Lemos, Liendo, Meineri, Sarkar '17]

$$\mathcal{F}(z,\bar{z},\sigma) \sim \sum_{\hat{\Delta},s} \int d^2z\, K_{\hat{\Delta},s}(z,\bar{z})\,\mathrm{Disc}\,\mathcal{F}(z,\bar{z},\sigma)$$

Inversion formula

The central tool is the inversion formula: [Caron-Huot; Lemos, Liendo, Meineri, Sarkar '17]

$$\mathcal{F}(z,\bar{z},\sigma) \sim \sum_{\hat{\Delta},s} \int d^2z\, K_{\hat{\Delta},s}(z,\bar{z})\, \mathrm{Disc}\, \mathcal{F}(z,\bar{z},\sigma)$$

The discontinuity is an imaginary part:

$$\mathrm{Disc}\, \mathcal{F}(z,\bar{z},\sigma) = \mathcal{F}(z,\bar{z}+i\epsilon,\sigma) - \mathcal{F}(z,\bar{z}-i\epsilon,\sigma)$$

Inversion formula

The central tool is the inversion formula: [Caron-Huot; Lemos, Liendo, Meineri, Sarkar '17]

$$\mathcal{F}(z,\bar{z},\sigma) \sim \sum_{\hat{\Delta},s} \int d^2z\, K_{\hat{\Delta},s}(z,\bar{z})\, \mathrm{Disc}\, \mathcal{F}(z,\bar{z},\sigma)$$

The discontinuity is an imaginary part:

$$\mathrm{Disc}\, \mathcal{F}(z,\bar{z},\sigma) = \mathcal{F}(z,\bar{z}+i\epsilon,\sigma) - \mathcal{F}(z,\bar{z}-i\epsilon,\sigma)$$

Main message:

- $\mathrm{Disc}\, \mathcal{F}$ reconstructs $\mathcal{F}$.

Inversion formula

The central tool is the inversion formula: [Caron-Huot; Lemos, Liendo, Meineri, Sarkar '17]

$$\mathcal{F}(z,\bar{z},\sigma) \sim \sum_{\hat{\Delta},s} \int d^2z\, K_{\hat{\Delta},s}(z,\bar{z})\, \mathrm{Disc}\, \mathcal{F}(z,\bar{z},\sigma)$$

The discontinuity is an imaginary part:

$$\mathrm{Disc}\, \mathcal{F}(z,\bar{z},\sigma) = \mathcal{F}(z,\bar{z}+i\epsilon,\sigma) - \mathcal{F}(z,\bar{z}-i\epsilon,\sigma)$$

Main message:

- $\mathrm{Disc}\, \mathcal{F}$ reconstructs $\mathcal{F}$.
- $\mathrm{Disc}\, \mathcal{F}$ is much simpler than $\mathcal{F}$.

Discontinuity

We use the bulk expansion of $\langle\!\langle\, \mathcal{O}_2\mathcal{O}_2 \,\rangle\!\rangle \propto \mathcal{F}$:

$$\mathcal{F} = \mathbb{1} + \lambda_{222} a_2 \, \mathcal{G}_{\mathcal{O}_2} + \sum_{\mathcal{O}} \lambda_{22\mathcal{O}} a_{\mathcal{O}} \, \mathcal{G}_{\mathcal{O}}$$

Discontinuity

We use the bulk expansion of $\langle\!\langle \mathcal{O}_2 \mathcal{O}_2 \rangle\!\rangle \propto \mathcal{F}$:

$$\mathcal{F} = \mathbb{1} + \lambda_{222} a_2 \, \mathcal{G}_{\mathcal{O}_2} + \sum_{\mathcal{O}} \lambda_{22\mathcal{O}} a_{\mathcal{O}} \, \mathcal{G}_{\mathcal{O}}$$

Physical input:

- Only single and double trace operators survive as $N \to \infty$.
- Unprotected single trace operators decouple as $\lambda \to \infty$.

Discontinuity

We use the bulk expansion of $\langle\!\langle\, \mathcal{O}_2 \mathcal{O}_2 \,\rangle\!\rangle \propto \mathcal{F}$:

$$\mathcal{F} = \mathbb{1} + \lambda_{222} a_2 \, \mathcal{G}_{\mathcal{O}_2} + \sum_{\mathcal{O}} \lambda_{22\mathcal{O}} a_{\mathcal{O}} \, \mathcal{G}_{\mathcal{O}}$$

Physical input:

- Only single and double trace operators survive as $N \to \infty$.
- Unprotected single trace operators decouple as $\lambda \to \infty$.

The sum is over double traces $\mathcal{O} \sim \mathcal{O}_2 \partial_{\mu_1} \ldots \partial_{\mu_\ell} \Box^n \mathcal{O}_2$: [Gonçalves '15]

$$\Delta_{\mathcal{O}} = 2\Delta_{\mathcal{O}_2} + \ell + 2n + \frac{1}{N^2} \left(a + \frac{b}{\lambda^{3/2}} + \ldots \right) + \ldots$$

Discontinuity

We use the bulk expansion of $\langle\!\langle \mathcal{O}_2\mathcal{O}_2 \rangle\!\rangle \propto \mathcal{F}$:

$$\mathcal{F} = \mathbb{1} + \lambda_{222}a_2\,\mathcal{G}_{\mathcal{O}_2} + \sum_{\mathcal{O}} \lambda_{22\mathcal{O}}a_{\mathcal{O}}\,\mathcal{G}_{\mathcal{O}}$$

Physical input:

- Only single and double trace operators survive as $N \to \infty$.
- Unprotected single trace operators decouple as $\lambda \to \infty$.

The sum is over double traces $\mathcal{O} \sim \mathcal{O}_2\partial_{\mu_1}\ldots\partial_{\mu_\ell}\Box^n\mathcal{O}_2$: [Gonçalves '15]

$$\Delta_{\mathcal{O}} = 2\Delta_{\mathcal{O}_2} + \ell + 2n + \frac{1}{N^2}\left(a + \frac{b}{\lambda^{3/2}} + \ldots\right) + \ldots$$

But we have $\mathrm{Disc}\,\mathcal{G}_{\mathcal{O}} \propto \gamma_{\mathcal{O}} = 0$ at the order we work!

Discontinuity

We use the bulk expansion of $\langle\!\langle\, \mathcal{O}_2\mathcal{O}_2 \,\rangle\!\rangle \propto \mathcal{F}$:

$$\mathcal{F} = \mathbb{1} + \lambda_{222} a_2\, \mathcal{G}_{\mathcal{O}_2} + \sum_{\mathcal{O}} \lambda_{22\mathcal{O}} a_{\mathcal{O}}\, \mathcal{G}_{\mathcal{O}}$$

Physical input:

- Only single and double trace operators survive as $N \to \infty$.
- Unprotected single trace operators decouple as $\lambda \to \infty$.

The sum is over double traces $\mathcal{O} \sim \mathcal{O}_2 \partial_{\mu_1} \dots \partial_{\mu_\ell} \Box^n \mathcal{O}_2$: [Gonçalves '15]

$$\Delta_{\mathcal{O}} = 2\Delta_{\mathcal{O}_2} + \ell + 2n + \frac{1}{N^2}\left(a + \frac{b}{\lambda^{3/2}} + \dots\right) + \dots$$

But we have $\mathrm{Disc}\, \mathcal{G}_{\mathcal{O}} \propto \gamma_{\mathcal{O}} = 0$ at the order we work!

So the discontinuity is dramatically simpler:

$$\mathrm{Disc}\, \mathcal{F} \sim \mathrm{Disc}\left(\mathbb{1} + \lambda_{222} a_2 \mathcal{G}_{\mathcal{O}_2}\right)$$

Final result

We have computed $\langle\!\langle\, \mathcal{O}_2 \mathcal{O}_2 \,\rangle\!\rangle^{(2)} \sim \sigma^2 F_0 + \sigma F_1 + F_2$ where:

$$F_0(z,\bar z) = -\frac{\sqrt{\lambda}}{2N^2}\frac{z\bar z}{(1-z)(1-\bar z)}\left[\frac{1+z\bar z}{(1-z\bar z)^2} + \frac{2z\bar z\log z\bar z}{(1-z\bar z)^3}\right]$$

$$F_1(z,\bar z) = \frac{\sqrt{\lambda}}{N^2}\left[\log(1+\sqrt{z\bar z}) + \frac{z\bar z}{(1-z\bar z)^2} \right.$$
$$\left. + \frac{z\bar z\big(5z\bar z - 2z^2\bar z^2 + z^3\bar z^3 - (z+\bar z)(2 - z\bar z + z^2\bar z^2)\big)\log z\bar z}{2(1-z)(1-\bar z)(1-z\bar z)^3}\right]$$

$$F_2(z,\bar z) = \frac{\sqrt{\lambda}}{8N^2}\left[-3 - \frac{2(z+\bar z)}{\sqrt{z\bar z}} + \frac{(z+\bar z)(1+z\bar z) - 4z\bar z}{(1-z\bar z)^2} \right.$$
$$+ \frac{2\big((z+\bar z)(1+z\bar z) - 4z\bar z\big)\log(1+\sqrt{z\bar z})}{z\bar z}$$
$$\left. + \frac{z\bar z\big((z+\bar z)(3 - 2z\bar z + z^2\bar z^2) - 6 + 6z\bar z - 4z^2\bar z^2\big)\log z\bar z}{(1-z\bar z)^3}\right]$$

Outlook

- We have computed $\langle\!\langle\, \mathcal{O}_L \mathcal{O}_L \,\rangle\!\rangle$ for length $L = 2, 3, 4$ CPOs.
  Can one guess a closed formula? Maybe in Mellin space?

Outlook

- We have computed $\langle\!\langle\, \mathcal{O}_L \mathcal{O}_L \,\rangle\!\rangle$ for length $L = 2, 3, 4$ CPOs.
  Can one guess a closed formula? Maybe in Mellin space?
- Unprotected CFT data $a_{\mathcal{O}}$ from integrability?

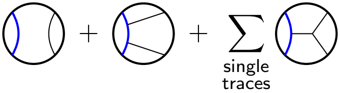
Outlook

- We have computed $\langle\!\langle\, \mathcal{O}_L \mathcal{O}_L \,\rangle\!\rangle$ for length $L = 2, 3, 4$ CPOs.
  Can one guess a closed formula? Maybe in Mellin space?

- Unprotected CFT data $a_{\mathcal{O}}$ from integrability?

- Explicit holographic calculation:

$$\langle\!\langle\, \mathcal{O}_L \mathcal{O}_L \,\rangle\!\rangle =$$



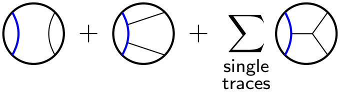
Outlook

- We have computed $\langle\!\langle\, \mathcal{O}_L \mathcal{O}_L \,\rangle\!\rangle$ for length $L = 2, 3, 4$ CPOs. Can one guess a closed formula? Maybe in Mellin space?

- Unprotected CFT data $a_{\mathcal{O}}$ from integrability?

- Explicit holographic calculation:

$$\langle\!\langle\, \mathcal{O}_L \mathcal{O}_L \,\rangle\!\rangle \; =$$

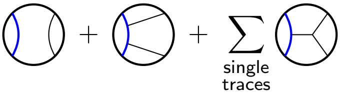


- Other holographic setups:
  - Half-BPS Wilson line in ABJM
  - Surface operators in $\mathcal{N} = 4$ SYM
  - M2- and M5-brane defects in $(2, 0)$ theory

Outlook

- We have computed $\langle\!\langle\, \mathcal{O}_L \mathcal{O}_L \,\rangle\!\rangle$ for length $L = 2, 3, 4$ CPOs.
  Can one guess a closed formula? Maybe in Mellin space?
- Unprotected CFT data $a_{\mathcal{O}}$ from integrability?
- Explicit holographic calculation:

$$\langle\!\langle\, \mathcal{O}_L \mathcal{O}_L \,\rangle\!\rangle =$$



- Other holographic setups:
  - Half-BPS Wilson line in ABJM
  - Surface operators in $\mathcal{N} = 4$ SYM
  - M2- and M5-brane defects in $(2, 0)$ theory
- Defects in condensed matter systems:
  - Monodromy defects [AGG, Liendo '21]
  - Quantum impurities (work in progress!)

Thank you!