

Bootstrapping holographic defect correlators in $\mathcal{N} = 4$ SYM

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Based on: Barrat, AGG, Liendo: 2108.13432.

The main characters

Stress-tensor ($20'$) multiplet

$$\mathcal{O}_2 = \text{tr} \phi^{\{i_1} \phi^{i_2\}}$$

$$\langle \mathcal{O}_2 \dots \mathcal{O}_2 \rangle$$

Wilson loop

$$\mathcal{W} = \frac{1}{N} \text{tr} P e^{\int d\tau (iA_\tau + \phi_6)}$$

$$\langle \mathcal{W}_{\text{circle}} \rangle = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$$

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The **simplest non-trivial** observable is

$$\langle\langle \mathcal{O}_2 \mathcal{O}_2 \rangle\rangle \equiv \frac{\langle \mathcal{W} \mathcal{O}_2 \mathcal{O}_2 \rangle}{\langle \mathcal{W} \rangle}$$

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•

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The expansion takes the form: [Giombi, Pestun '12]

$$\langle\langle \mathcal{OO} \rangle\rangle = \langle\langle \mathcal{OO} \rangle\rangle^{(0)} + \frac{\lambda}{N^2} \left(\langle\langle \mathcal{OO} \rangle\rangle^{(1)} + \frac{1}{\sqrt{\lambda}} \langle\langle \mathcal{OO} \rangle\rangle^{(2)} + \dots \right) + \dots$$

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The diagrams represent Feynman diagrams for the two-point correlation function $\langle\langle \mathcal{OO} \rangle\rangle$ in planar $\mathcal{N} = 4$ SYM. Each diagram is a circle with a blue arc on the left side representing an operator insertion. Diagram 1 is a bubble diagram with a vertical line connecting the two vertices on the right. Diagram 2 is a bubble diagram with a vertical line and a diagonal line connecting the two vertices on the right. Diagram 3 is a bubble diagram with a vertical line and two diagonal lines connecting the two vertices on the right.

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The diagrams are Feynman diagrams for the two-point function $\langle\langle \mathcal{OO} \rangle\rangle$. The first diagram is a circle with a blue arc on the left and a white arc on the right. The second diagram is a circle with a blue arc on the left and a white arc on the right, with a vertical line connecting the two arcs. The third diagram is a circle with a blue arc on the left and a white arc on the right, with a vertical line connecting the two arcs and a horizontal line connecting the two arcs.

In this work:

- ▶ Focus on the leading non-trivial correction $\langle\langle \mathcal{OO} \rangle\rangle^{(2)}$
- ▶ We **do not** calculate the Witten diagram
- ▶ We **bootstrap** the result with an **inversion formula**

Crossing equation

Our two-point function is

$$\langle\langle \mathcal{O}_2 \mathcal{O}_2 \rangle\rangle \propto \mathcal{F}(\underbrace{z, \bar{z}}_{\substack{\text{spacetime} \\ \text{cross-ratios}}, \sigma) = \underbrace{\sigma^2 F_0 + \sigma F_1 + F_2}_{R\text{-symmetry channels}}$$

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Bulk-bulk : $\mathcal{O}_2 \times \mathcal{O}_2 \sim \sum_{\mathcal{O}} \lambda_{22\mathcal{O}} \mathcal{O}$

Bulk-defect : $\mathcal{O}_2 \sim \sum_{\hat{\mathcal{O}}} b_{2\hat{\mathcal{O}}} \hat{\mathcal{O}}$

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Our correlator $\langle\langle \mathcal{O}_2 \mathcal{O}_2 \rangle\rangle$ contains an infinite amount of CFT data

$$\mathcal{F}(z, \bar{z}, \sigma) = \sum_{\mathcal{O}} \lambda_{22\mathcal{O}} a_{\mathcal{O}} \mathcal{G}_{\mathcal{O}}(z, \bar{z}, \sigma) = \sum_{\hat{\mathcal{O}}} b_{2\hat{\mathcal{O}}}^2 \hat{\mathcal{G}}_{\hat{\mathcal{O}}}(z, \bar{z}, \sigma)$$

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Reverse logic: consistency with crossing fixes \mathcal{F} !

Inversion formula

The central tool is the **inversion formula**: [Caron-Huot; Lemos, Liendo, Meineri, Sarkar '17]

$$\mathcal{F}(z, \bar{z}, \sigma) \sim \sum_{\hat{\Delta}, s} \int d^2 z K_{\hat{\Delta}, s}(z, \bar{z}) \text{Disc } \mathcal{F}(z, \bar{z}, \sigma)$$

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The discontinuity is an **imaginary part**:

$$\text{Disc } \mathcal{F}(z, \bar{z}, \sigma) = \mathcal{F}(z, \bar{z} + i\epsilon, \sigma) - \mathcal{F}(z, \bar{z} - i\epsilon, \sigma)$$

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Main message:

- ▶ Disc \mathcal{F} **reconstructs** \mathcal{F} .
- ▶ Disc \mathcal{F} is **much simpler** than \mathcal{F} .

Discontinuity

We use the bulk expansion of $\langle\langle \mathcal{O}_2 \mathcal{O}_2 \rangle\rangle \propto \mathcal{F}$:

$$\mathcal{F} = \mathbb{1} + \lambda_{222} a_2 \mathcal{G}_{\mathcal{O}_2} + \sum_{\mathcal{O}} \lambda_{22\mathcal{O}} a_{\mathcal{O}} \mathcal{G}_{\mathcal{O}}$$

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The sum is over **double traces** $\mathcal{O} \sim \mathcal{O}_2 \partial_{\mu_1} \dots \partial_{\mu_\ell} \square^n \mathcal{O}_2$: [Gonçalves '15]

$$\Delta_{\mathcal{O}} = 2\Delta_{\mathcal{O}_2} + \ell + 2n + \frac{1}{N^2} \left(a + \frac{b}{\lambda^{3/2}} + \dots \right) + \dots$$

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So the discontinuity is **dramatically simpler**:

$$\text{Disc } \mathcal{F} \sim \text{Disc} \left(\mathbb{1} + \lambda_{222} a_2 \mathcal{G}_{\mathcal{O}_2} \right)$$

Final result

We have computed $\langle\langle \mathcal{O}_2 \mathcal{O}_2 \rangle\rangle^{(2)} \sim \sigma^2 F_0 + \sigma F_1 + F_2$ where:

$$F_0(z, \bar{z}) = -\frac{\sqrt{\lambda}}{2N^2} \frac{z\bar{z}}{(1-z)(1-\bar{z})} \left[\frac{1+z\bar{z}}{(1-z\bar{z})^2} + \frac{2z\bar{z} \log z\bar{z}}{(1-z\bar{z})^3} \right]$$

$$F_1(z, \bar{z}) = \frac{\sqrt{\lambda}}{N^2} \left[\log(1 + \sqrt{z\bar{z}}) + \frac{z\bar{z}}{(1-z\bar{z})^2} + \frac{z\bar{z}(5z\bar{z} - 2z^2\bar{z}^2 + z^3\bar{z}^3 - (z+\bar{z})(2-z\bar{z} + z^2\bar{z}^2)) \log z\bar{z}}{2(1-z)(1-\bar{z})(1-z\bar{z})^3} \right]$$

$$F_2(z, \bar{z}) = \frac{\sqrt{\lambda}}{8N^2} \left[-3 - \frac{2(z+\bar{z})}{\sqrt{z\bar{z}}} + \frac{(z+\bar{z})(1+z\bar{z}) - 4z\bar{z}}{(1-z\bar{z})^2} + \frac{2((z+\bar{z})(1+z\bar{z}) - 4z\bar{z}) \log(1 + \sqrt{z\bar{z}})}{z\bar{z}} + \frac{z\bar{z}((z+\bar{z})(3-2z\bar{z} + z^2\bar{z}^2) - 6 + 6z\bar{z} - 4z^2\bar{z}^2) \log z\bar{z}}{(1-z\bar{z})^3} \right]$$

Outlook

- ▶ We have computed $\langle\langle \mathcal{O}_L \mathcal{O}_L \rangle\rangle$ for length $L = 2, 3, 4$ CPOs.
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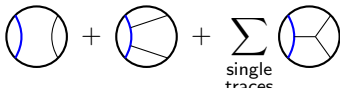
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$$\langle\langle \mathcal{O}_L \mathcal{O}_L \rangle\rangle = \text{[Diagram 1]} + \text{[Diagram 2]} + \sum_{\text{single traces}} \text{[Diagram 3]}$$

The equation shows the correlator $\langle\langle \mathcal{O}_L \mathcal{O}_L \rangle\rangle$ as a sum of three terms. The first term is a circle with a blue arc on the left. The second term is a circle with a blue arc on the left and two lines extending from the right side. The third term is a circle with a blue arc on the left and three lines extending from the right side, with a summation symbol \sum and the text "single traces" below it.

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 - ▶ Half-BPS Wilson line in ABJM
 - ▶ Surface operators in $\mathcal{N} = 4$ SYM
 - ▶ M2- and M5-brane defects in $(2, 0)$ theory

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- ▶ Defects in **condensed matter** systems:
 - ▶ Monodromy defects [AGG, Liendo '21]
 - ▶ Quantum impurities (work in progress!)

Thank you!