An Exact AdS_3/CFT_2 correspondence

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Outline

1. The basic setup
2. Mapping of the Hilbert space
3. Mapping of correlators
4. Black holes
5. Connections to integrability

AdS_3/CFT_2

[Maldacena '97, Larsen & Martinec '99, ...]

strings on $AdS_3 \times S^3 \times T^4$ $\longleftrightarrow$ (deformation of) $\mathrm{Sym}^N(T^4)$

- Both sides possess a 20-dimensional moduli space
- The "symmetric orbifold point" is a special point in moduli space
- On the CFT side only the symmetric orbifold is under control
- The dual string background has some special flux combination

What is it?

The tensionless point

- The symmetric orbifold possesses higher spin fields [Gaberdiel, Gopakumar '14]

  ⇒ Bulk theory needs unbroken higher spin symmetry

  ⇒ Tensionless limit of string theory

  ⇒ Low number of flux quanta

- It turns out that one unit of NS-NS flux & no R-R flux seems to work. [(LE), Gaberdiel, Gopakumar '18 ; Giribet, Hull, Kleban, Porrati, Rabinovici '18] $N \sim \frac{1}{g_s^2}$

strings on $AdS_3 \times S^3 \times T^4$ with one unit of NS-NS flux	$\overset{?}{=}$	$\mathrm{Sym}^N(T^4)$

Features

- Usually pure NS-NS flux backgrounds have a long string continuum.

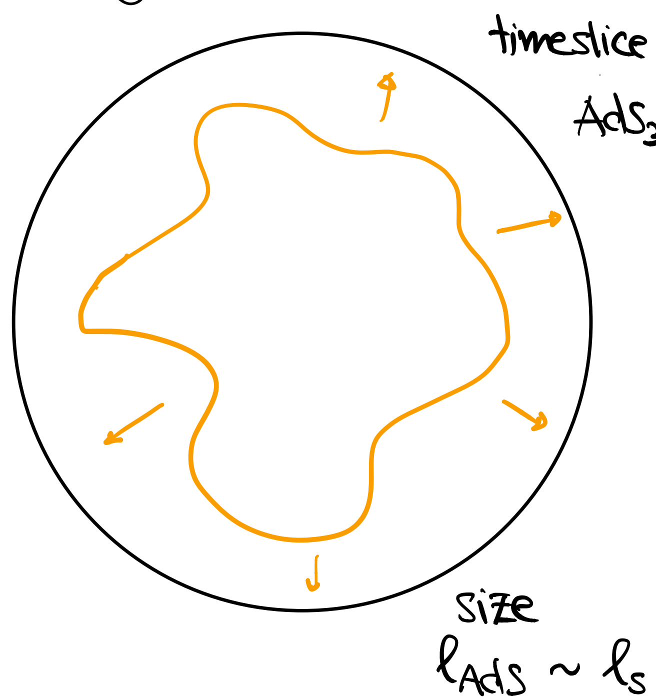
  Radial mode in the picture

- For one unit of flux, this continuum disappears. [LE, Gaberdiel, Gopakumar '18]

  $\Rightarrow$ Expect nice spacetime CFT

- Tension is small

  $\Rightarrow$ Long strings become the generic states of the theory

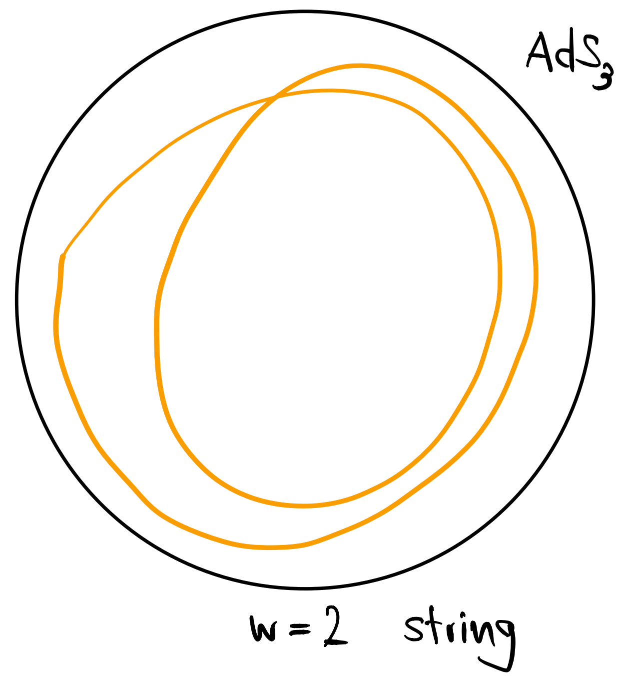


Winding strings

- Strings can wind around AdS_3
- Even though AdS_3 is contractible, this is a well-defined integer $w \geq 1$ in the quantum theory.

a.k.a. spectral flow

[Maldacena, Ooguri 00']



The symmetric orbifold

- The symmetric orbifold of T^4 is an orbifold of a free theory

$$\mathrm{Sym}^N(T^4) = \frac{T^4 \times T^4 \times \ldots \times T^4}{S_N}$$

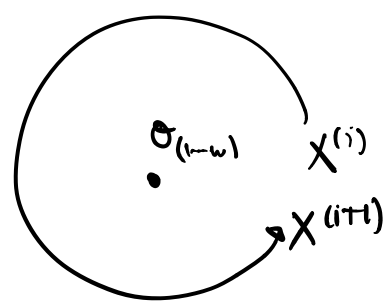
$\Rightarrow$ Non trivial but solvable

- Important fields are twist fields

$$\sigma_w = \sigma_{[(1\cdots w)]} = \sum_{g\in S_N} \sigma_{g(1\cdots w)g^{-1}}$$

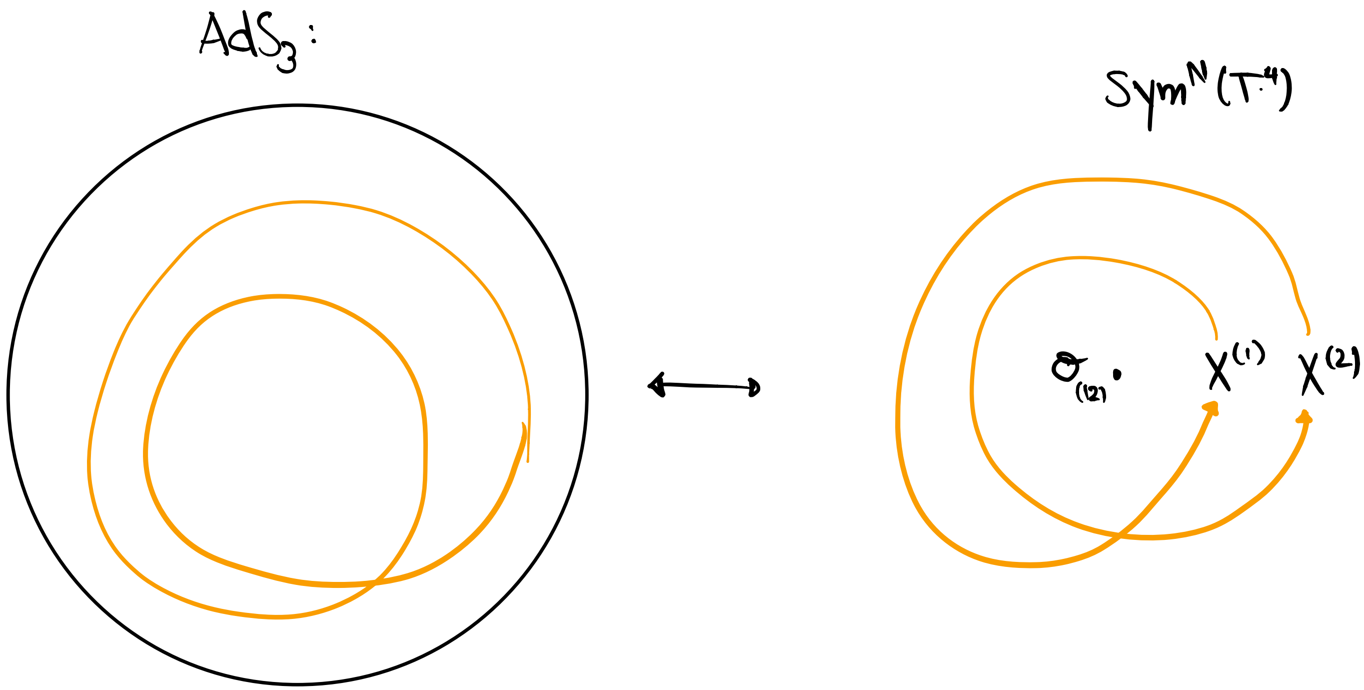
$\Rightarrow$ Introduce monodromy

- Single cycle twist fields correspond to single string states in the bulk



$i = 1, \ldots, w.$

Mapping of the Hilbert space



- $X^{(1)}$ & $X^{(2)}$ are the sheets of a Riemann surface

- This covering surface of the boundary is identified with the worldsheet in AdS_3!

The spectrum

- The worldsheet theory can be formulated exactly in terms of a super WZW model [Berkovits, Vafa, Witten '99]
  - $\Rightarrow$ Can compute full bulk spectrum (exact in α', small g_s)
  - $\Rightarrow$ Matches with single particle spectrum of $\mathrm{Sym}^N(T^4)$ in large N limit [LE, Gaberdiel, Gopakumar '18]
- Gives a dictionary between (perturbative) states on both sides

Worldsheet correlators

- Since we know the mapping of Hilbert spaces, we can study correlators.

Worldsheet vertex operator

$$V^{w}(x, z) \longleftrightarrow \sigma_w(x)$$

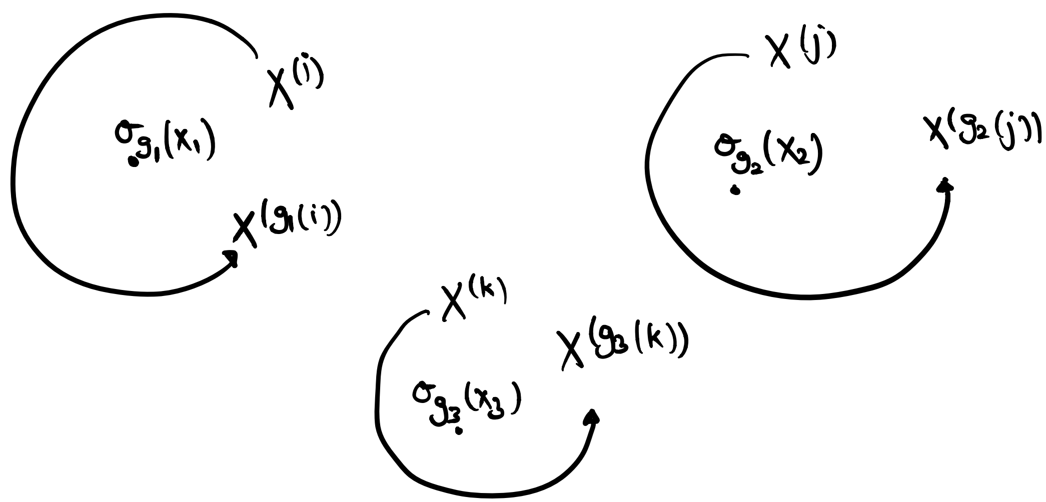
winding, boundary coordinate (Euclidean), worldsheet coordinate

$$\sum_{g=0}^{\infty} g_s^{2g-2+n} \int_{\mathcal{M}_{g,n}} \left\langle \prod_{i=1}^{n} V^{w_i}(x_i, z_i) \right\rangle_{\Sigma_g} \stackrel{?}{=} \left\langle \prod_{i=1}^{n} \sigma_{w_i}(x_i) \right\rangle_{S^2}$$

Correlators in $\mathrm{Sym}^N(T^4)$

Compute $\langle \prod_i \sigma_{w_i}(x_i) \rangle$.

$$\sigma_w = \sum_{g \in S_N} \sigma_{g(1\cdots w)g^{-1}}$$



- Some of the $X^{(i)}$'s have monodromy
- They define the sheets of a branched cover

$$\Gamma : \Sigma_g \longrightarrow S^2.$$

Correlators of $\mathrm{Sym}^N(T^4)$ (cont.)

- Instead of computing the correlator on S^2 one can perform a conformal transformation to Σ_g.

$$\left\langle \prod_{i=1}^{n} \sigma_{w_i}(x_i) \right\rangle = \sum_{\substack{\text{branched} \\ \text{covers } \Sigma_g}} e^{-S_L[\phi]} \left\langle \prod_{i=1}^{n} \tilde{\sigma}(z_i) \right\rangle_{\Sigma_g}$$

Liouville action evaluated on Weyl factor of conformal transformation

- $S_L[\phi]$ can be evaluated explicitly in terms of the corresponding covering map. [Lunin, Mathur '00]

Match with worldsheet

- Compare

$$\sum_{g=0}^{\infty} g_s^{2g-2+n} \int_{\mathcal{M}_{g,n}} \left\langle \prod_{i=1}^{n} V^{w_i}(x_i, z_i) \right\rangle_{\Sigma_g} \overset{?}{=} \sum_{g=0}^{\infty} N^{-g+1-\frac{n}{2}} \sum_{\substack{\text{branched} \\ \text{covers } \Sigma_g}} e^{-S_L[\phi]} \left\langle \prod_{i=1}^{n} \tilde{\sigma}(z_i) \right\rangle_{\Sigma_g}$$

- Same idea as for the spectrum [Pakman, Rastelli, Razamat '09]

  the worldsheet is the covering space

- For this to work $\int_{\mathcal{M}_{g,n}}$ has to localize:

$$\int_{\mathcal{M}_{g,n}} \longrightarrow \sum_{\substack{\text{branched} \\ \text{covers } \Sigma_g}}$$

Localization

- Localization of worldsheet correlators

$$\left\langle \prod_{i=1}^{n} V^{w_i}(x_i, z_i) \right\rangle$$

  is not very transparent, since the covering map does not appear in the definition of the worldsheet theory.

- Evidence for localization was given in
  [LE, Gaberdiel, Gopakumar '19; LE '20]

- A proof for $g=0$ was given in
  [Dei, Gaberdiel, Gopakumar, Knighton '20]

  $\Rightarrow$ More direct understanding missing!

Boundary torus partition function

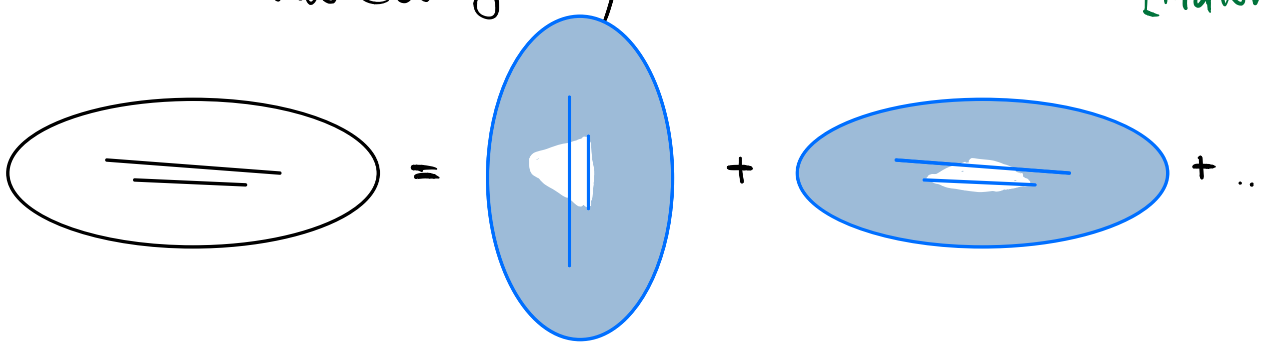
- What about non-perturbative objects such as black holes?

  ⇒ Where are they in the CFT Hilbert space?

- Let's try to compute the $\mathrm{Sym}^N(T^4)$ torus partition function from the bulk.

- In semiclassical gravity:

for 3d gravity:
[Maloney, Witten '07]



thermal AdS_3

Euclidean BTZ black hole

other geometries

Thermal AdS_3 partition function

- Study this tensionless string on thermal AdS_3

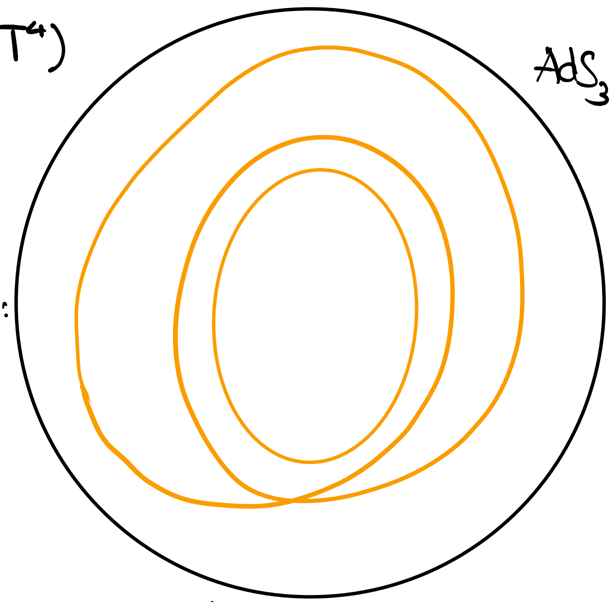
  thermal AdS_3 = Euclidean global $AdS_3 / \mathbb{Z}$

  $\Rightarrow$ worldsheet theory for thermal AdS_3 is a $\mathbb{Z}$-orbifold of global AdS_3 theory

$\Rightarrow$ Can compute 1-loop string partition function exactly.

What does string perturbation theory compute?

- $w=2$ & $w=1$ string: state in $\mathrm{Sym}^3(T^4)$



- In string perturbation theory, the number of strings is not fixed: grand canonical ensemble

- In $\mathrm{Sym}^N(T^4)$, N corresponds to the number of fundamental strings in the background: canonical ensemble

$\to$ Compute grand canonical partition function

$$\mathcal{Z}(\sigma) = \sum_{N=0}^{\infty} e^{2\pi i N \sigma} Z_{\mathrm{Sym}^N(T^4)}$$

σ: chemical potential

Thermal AdS_3 partition function (cont.)

- The chemical potential can be incorporated into the worldsheet theory [Kim, Porrati '15]

$$Z_{ws}(\tau_{bdry}, \tau_{ws}, \sigma) = \sum_{a,b,c,d \in \mathbb{Z}} \tfrac{1}{2} \mathrm{Im}\,\tau_{bdry}\, \delta^2\big(\tau_{bdry}(c\tau_{ws}+d) - a\tau_{ws} - b\big)\, e^{2\pi i \sigma \det\begin{pmatrix} a & b \\ c & d\end{pmatrix}}\, Z^{T^4}\begin{bmatrix} a/2 \\ b/2 \end{bmatrix}(\tau_{ws})$$

τ_{bdry}: boundary modular parameter

τ_{ws}: worldsheet modular parameter

$Z^{T^4}\begin{bmatrix} a/2 \\ b/2 \end{bmatrix}$: torus partition function of T^4 σ-model with spin structure $\begin{bmatrix} a/2 \\ b/2 \end{bmatrix}$

- Worldsheet partition function again localizes on

$$\tau_{bdry} = \frac{a\tau_{ws} + b}{c\tau_{ws} + d}$$

→ For these modular parameters there is an unramified cover

$$\Gamma : T^2_{ws} \longrightarrow T^2_{bdry} \quad \text{of degree} \quad \det\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Thermal AdS_3 partition function (cont.)

- Presence of δ^2 makes $\int_{\mathcal{M}_1}$ easy:

$$\mathcal{Z}_{\text{thermal } AdS_3}(\tau_{bdry}, \sigma) = \exp\left[\int_{\mathcal{F}} \frac{d^2\tau_{ws}}{\operatorname{Im}\tau_{ws}} Z_{ws}(\tau_{bdry}, \tau_{ws}, \sigma)\right]$$

$$= \exp\left[\sum_{a,d=1}^{\infty} \sum_{b\in\mathbb{Z}_a} \frac{p^{ad}}{ad} Z^{T^4}\begin{bmatrix} b/2 \\ a/2 \end{bmatrix}\left(\frac{d\tau_{bdry}+b}{a}\right)\right]$$

$$= \mathcal{Z}_{Sym}(\tau_{bdry}, \sigma).$$

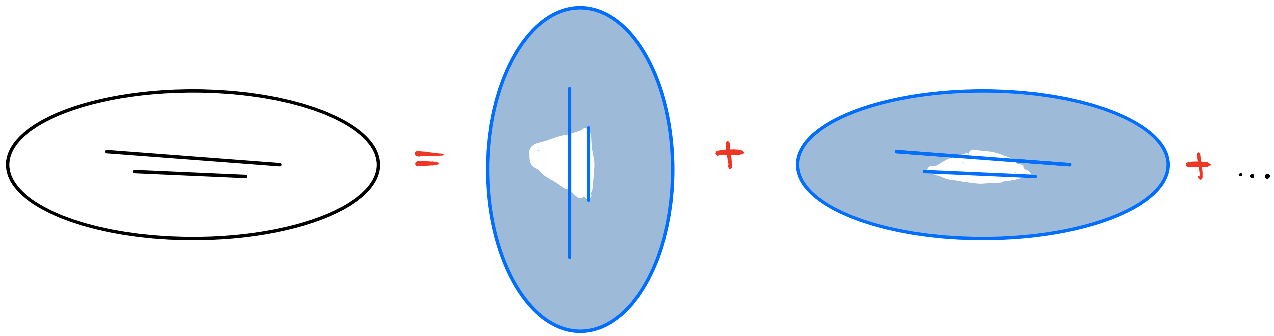
- Similar calculations (for same spin structure)

$$\mathcal{Z}_{\text{thermal } AdS_3} = \mathcal{Z}_{\text{BTZ black hole}} = \mathcal{Z}_{\text{conical defect}} = \mathcal{Z}_{Sym}$$

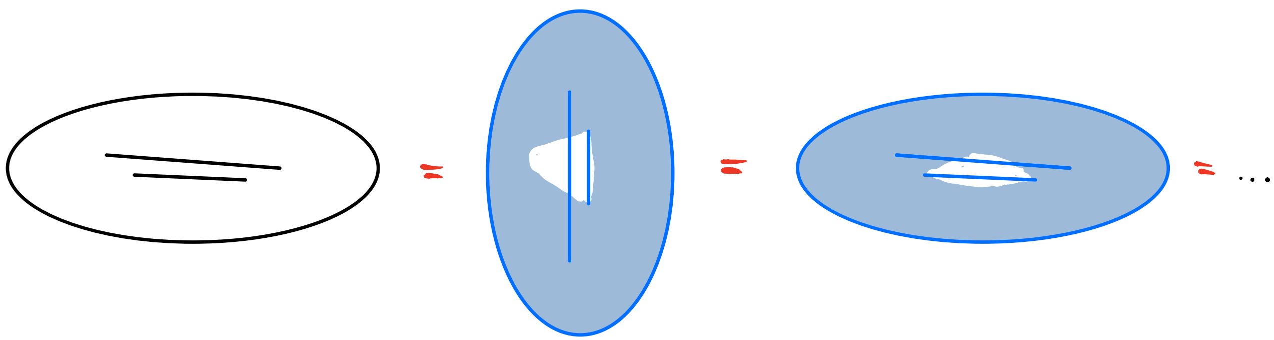
[LE '20]

Sum over geometries ?

- Sum over geometries not needed. Instead of



we have



String calculation ok?

There are several possible loopholes in the computation:

- We did only a 1-loop computation
  Localization principle predicts absence of higher genus corrections, but it would be good to show this directly
- D-instanton corrections?
- Backreaction? We consider very long strings and they can become heavy

We will assume in the following that we can trust the computation.

Stringy black holes

- We can thus identify the BTZ black hole as a highly excited perturbative string state in thermal AdS_3

$$\boxed{\text{BTZ black hole} = \begin{array}{l}\text{single long string winding } N \text{ times} \\ \text{around thermal } AdS_3\end{array}}$$

$\Rightarrow$ Black hole/string transition [Susskind '93; Horowitz, Polchinski '96; Giveon, Kutasov, Rabinovici, Sever '05]

- Hawking-Page transition becomes a transition between many short strings & the single long string

Worldsheet integrability?

- So far most of the computations have been performed using worldsheet CFT

+	−
- Good control over representations	- Winding string correlators complicated
- Good to discuss higher genus	- logarithmic CFT
- Complex structure of the worldsheet manifest	- Complicated BRST structure (not fully solved)
- Easy to consider other backgrounds such as thermal AdS_3	- Localization not transparent

Worldsheet integrability? (cont.)

- Integrability is catching up!
- An S-matrix for pure NS-NS backgrounds was proposed by [Baggio, Sfondrini '18; Dei, Sfondrini '18] that reproduces the full spacetime spectrum (including long strings [Sfondrini '20])
- TBA can be solved in closed form
- This reproduces again the spectrum of the symmetric orbifold
- The computation is much simpler than the one in worldsheet CFT

Correlation functions from integrability?

- Next step would be correlation functions.
- Answers are "simple"

$\Rightarrow$ Testing ground for the hexagon proposal

- Learn how to deal with higher point functions.
- Localization more obvious from integrability ?

Summary

$\boxed{AdS_3 \times S^3 \times T^4 \text{ with one unit of NS-NS flux} \;=\; \mathrm{Sym}^N(T^4)}$

- Spectrum can be matched
- Correlators in $\mathrm{Sym}^N(T^4)$ can be computed by passing to a ramified cover which is identified with the worldsheet
- Worldsheet correlators localize in $\mathcal{M}_{g,n}$ to all ramified covers
- The spectrum has only perturbative excitations and a black hole is identified with a single long string