The Wilson Loop — Large Spin Dictionary

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LIJC

Gong Show

Outline

- Define correlation function
- Correlation $\longleftrightarrow$ WL
- Derivation
- Integrability

1. What we consider:

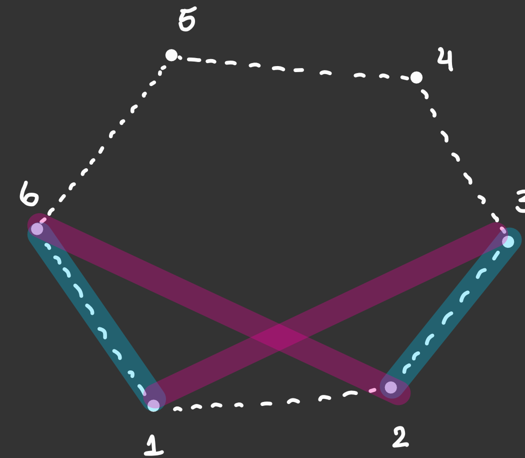
- 6pt functions
- lightest scalars (20')
- planar limit

3. Double light-cone limit:

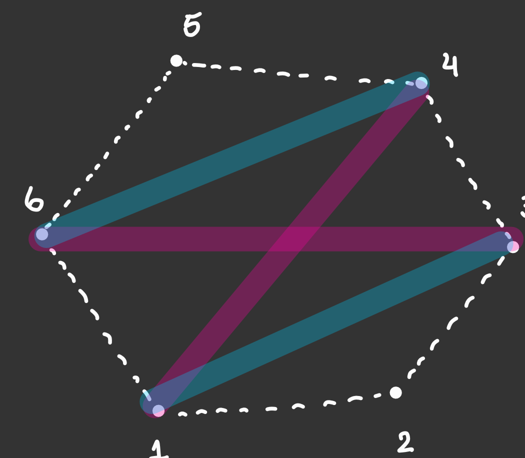
- $x^2_{i,i+1} \to 0$
- $v_i \to 0$
- $U_i \to$ finite
- non-trivial

2. Cross-ratios

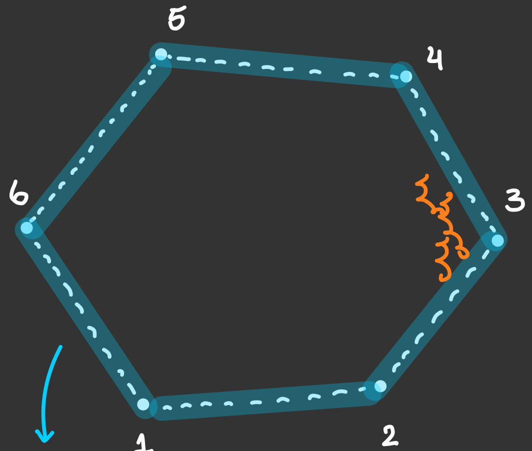
- $v_1 = \dfrac{x^2_{16}\, x^2_{23}}{x^2_{26}\, x^2_{13}}$
- $v_{i+1} = v_i \Big|_{x_i \to x_{i+1}}$
- 6 v's



- $U_1 = \dfrac{x^2_{13}\, x^2_{46}}{x^2_{14}\, x^2_{36}}$
- $U_{i+1} = U_i \Big|_{x_i \to x_{i+1}}$
- 3 U's



Correlation function:



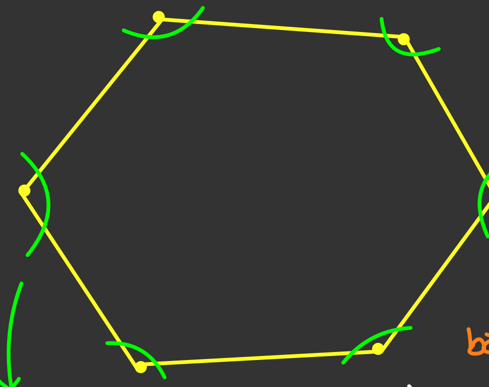
diverge ($\frac{1}{x^2} \to \infty$)

very fast particle

interacts with the background gauge field

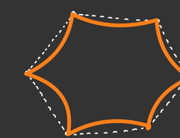
Wilson line

Wilson loop:



cusp divergences

back reaction

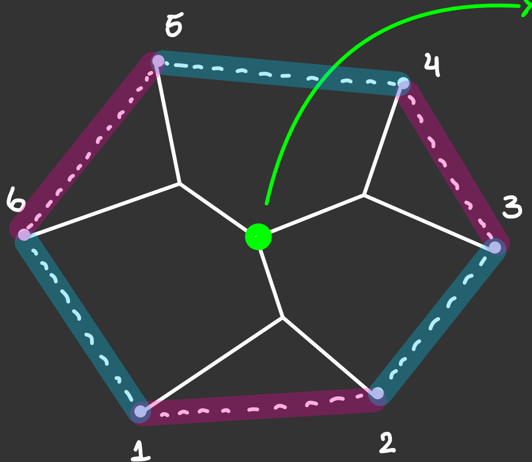


Duality:

$$G_6 = \text{Sudakov} \times \text{Recoil} \times W(u_1, u_2, u_3)$$

[1007.3243]

Correlation function in the double light-cone limit:



$$\langle \mathcal{O}_{J_1}(x_1,\epsilon_1), \mathcal{O}_{J_2}(x_2,\epsilon_2), \mathcal{O}_{J_3}(x_3,\epsilon_3)\rangle = \frac{\sum_{\ell_i} C^{J_1,J_2,J_3}_{\ell_1,\ell_2,\ell_3} V^{J_1-\ell_2-\ell_3}_{1,23} V^{J_2-\ell_3-\ell_1}_{2,31} V^{J_3-\ell_1-\ell_2}_{3,12} H^{\ell_1}_{23} H^{\ell_2}_{31} H^{\ell_3}_{12}}{(x_{12}^2)^{\frac{\kappa_1+\kappa_2-\kappa_3}{2}} (x_{23}^2)^{\frac{\kappa_2+\kappa_3-\kappa_1}{2}} (x_{31}^2)^{\frac{\kappa_3+\kappa_1-\kappa_2}{2}}},$$

$$= \sum_{\Delta_i} \sum_{J_i} \sum_{\ell_i} C^{\bullet\bullet\bullet}(J_i,\ell_i)\, \mathcal{F}(v_i, U_i)$$

$$\underset{\tau=2}{=} \sum_{J_i} \sum_{\ell_i} C^{\bullet\bullet\bullet}(J_i,\ell_i)\, \mathcal{F}(v_i, U_i)$$

$$\underset{\tau=2}{=} \int dJ_i\, d\ell_i\, C^{\bullet\bullet\bullet}(J_i,\ell_i)\, \mathcal{F}(v_i, U_i)$$

$$\underset{\tau=2}{=} \int dJ_i\, C^{\bullet\bullet\bullet}(J_i,\ell_i)\, \mathcal{F}(v_i, U_i)$$

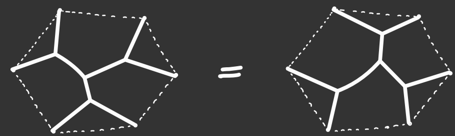
$$\simeq G_6$$

large spin structure cte.

$(v_i \to 0)$ simplified conformal block

- Snowflake OPE
- $x_{12}^2, x_{34}^2, x_{56}^2 \to 0$: leading twist
- $x_{23}^2, x_{45}^2, x_{61}^2 \to 0$: large spin
- saddle point

Bootstrap

- 
- cyclicity
- constrains : $C^{\bullet\bullet\bullet}(J_i, \ell_i)$
- for $N=4$ it fixes.

To be complete:

- x_i definition

$$x_1 = \frac{J_2 J_3}{J_1}\sqrt{\frac{v_2 v_4}{v_6}}, \qquad x_3 = \frac{J_1 J_2}{J_3}\sqrt{\frac{v_2 v_6}{v_4}}, \qquad x_5 = \frac{J_1 J_3}{J_2}\sqrt{\frac{v_4 v_6}{v_2}},$$

- Anomalous dimension

$$\gamma \simeq f(\lambda)\ln(J) + g(\lambda)$$

Results :

- G_6 = Sudakov × Recoil × $\mathbb{W}(U_1, U_2, U_3)$

$$\widetilde{G}_6 = \underbrace{\mathbb{W}(U_1,U_2,U_3)}_{\text{Renormalized Wilson loop}} \underbrace{\times \exp\Big(\sum_i \frac{f}{16}\ln v_i \ln v_{i+3} - \frac{f}{8}\ln v_i \ln v_{i+1} + \frac{g - f\gamma_E}{4}\ln v_i\Big) \times}_{\text{Sudokov Factor}} \tag{22}$$

$$\times \underbrace{\mathcal{N}\left(\int_0^\infty \prod_{j=1}^{6} dx_j \, e^{-\sum_{i=1}^{6}\left(x_i + \frac{g}{2}\mathbf{ln}(x_i) - \frac{f}{4}\mathbf{ln}(x_i)\mathbf{ln}(x_{i+1}) - \frac{f}{4}\mathbf{ln}(x_i)\mathbf{ln}(v_i)\right) + \sum_i \frac{f\gamma_E}{4}\mathbf{ln}(v_i)}\right)}_{\text{Recoil } J},$$

where $\mathbf{ln}(x) = \ln(x) + \gamma_E$.

- Large spin structure cte ↔ ren. Wilson loop

$$\hat{C}^{\bullet\bullet\bullet}(J_i, \ell_i) = \underbrace{\mathcal{N}\prod_{i=1}^{3}\left(\frac{J_i \ell_i}{2\ell_{i+1}\ell_{i-1}}\right)^{\frac{\gamma_i}{2}}}_{\text{conversion factor}} \times \mathbb{W}(U_1, U_2, U_3).$$

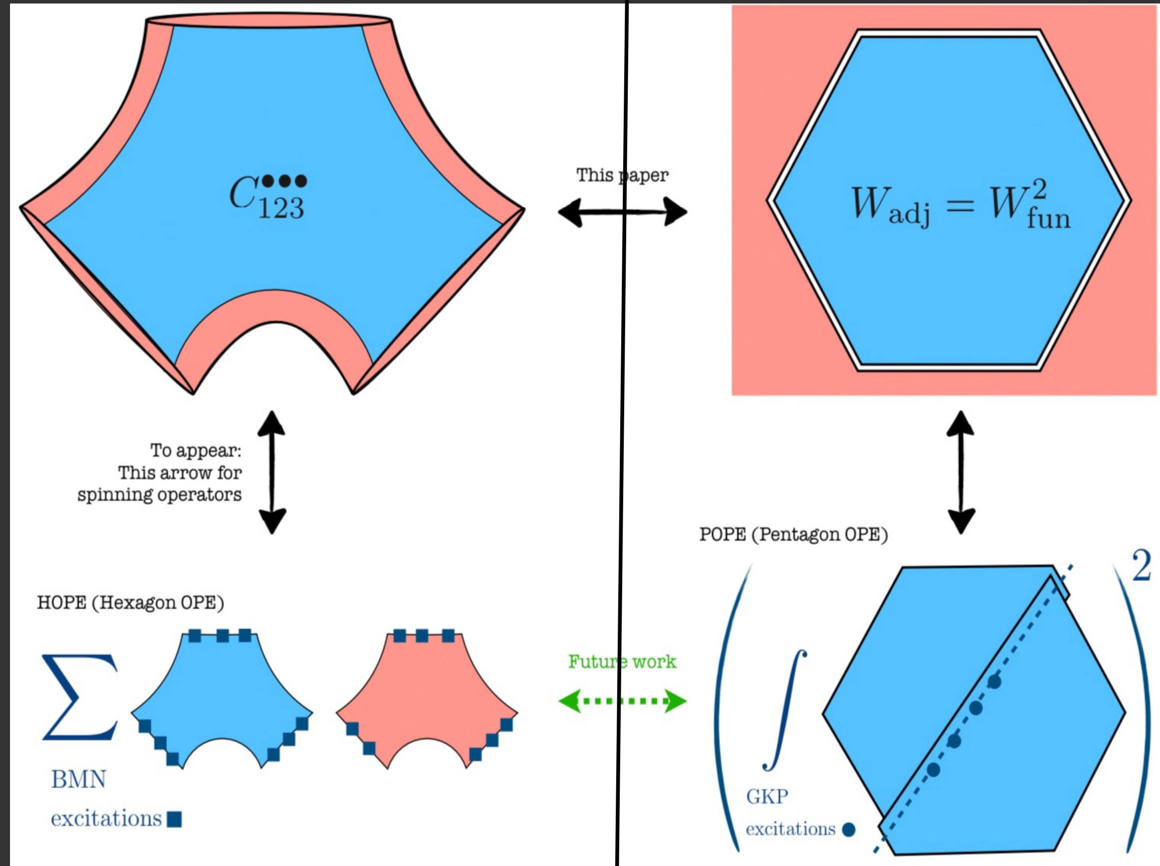
↳ conjectured [1006.2788]

- Dictionary : $\ell(U)$

$$\ell_1 = \frac{J_2 J_3}{J_2 + J_3 + J_1\sqrt{\frac{U_2}{U_1 U_3}}}, \qquad \ell_2 = \frac{J_1 J_3}{J_1 + J_3 + J_2\sqrt{\frac{U_1}{U_2 U_3}}}, \qquad \ell_3 = \frac{J_1 J_2}{J_1 + J_2 + J_3\sqrt{\frac{U_3}{U_1 U_2}}}$$

Integrability.

- BMN ↔ GKP
- Hexagon ↔ Pentagon
- Close ↔ open
- Unifying description



[1505.06745] [1303.1396]