Hidden Symmetry in 4d $\mathcal{N}$=2 SCFTs

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Based on work with Enrico Andriolo, Hanno Bertle, Elli Pomoni, Konstantinos Zoubos (to appear soon)

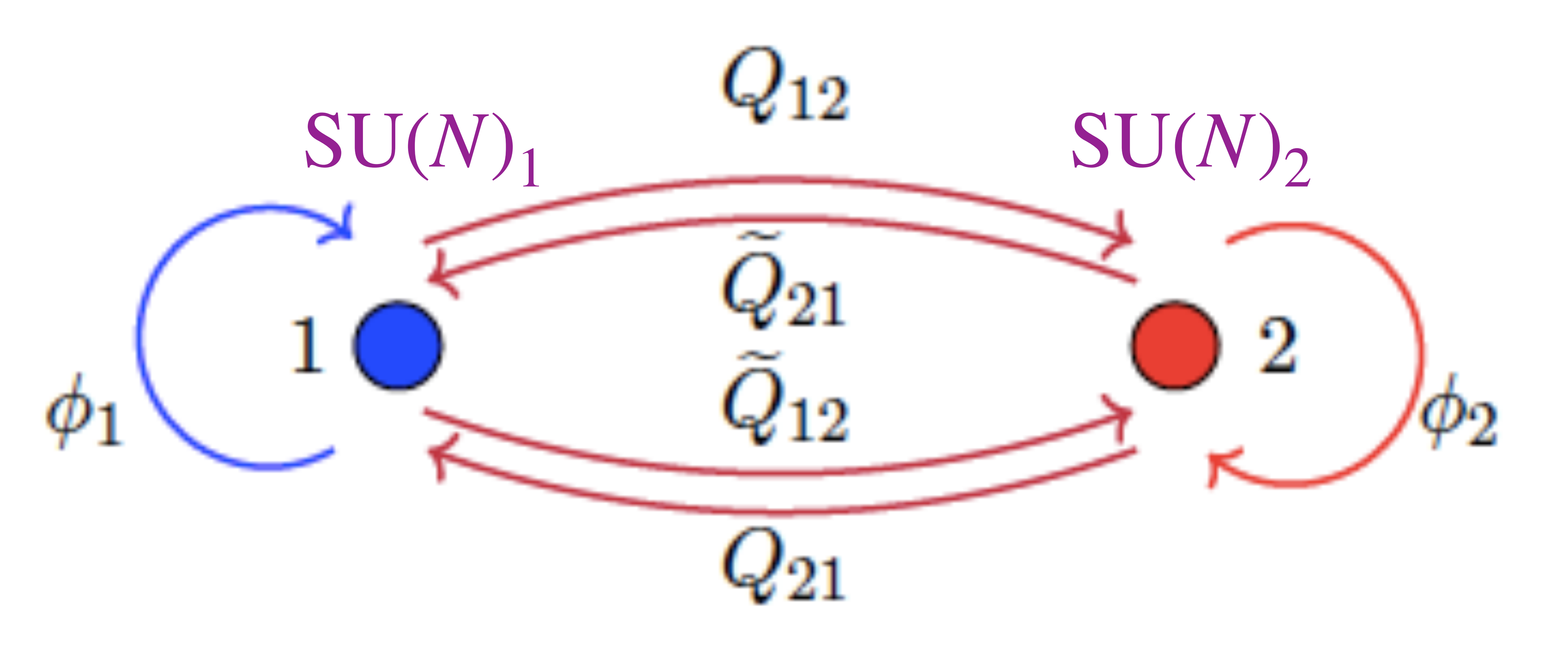
Motivation

- Symmetries play an important role in modern theoretical physics. It has been known for decades that the notion of Lie groups/algebras is insufficient to describe all continuous symmetries. A more flexible and powerful treatment is based on (quasi-)Hopf algebras.
[Drinfeld 1989]
- A large class of $\mathcal{N} = 2$ SCFTs were recently conjectured to be integrable in the planar limit, and a quantum group $\mathrm{PSU}(2,2|4)_\kappa$ shows up in the one-loop spectral problem.
[Pomoni, Rabe, Zoubos 2021]
- Aim: have a better understanding of the quantum symmetry underlying the $\mathcal{N} = 2$ SCFTs.

Simple example: $\mathcal{N} = 2$ SCFT of $\hat{A}_1$ quiver

- Can be obtained from the $\mathcal{N} = 4$ SYM theory by a $\mathbb{Z}_2$ orbifold projection + an exactly marginal deformation.

- String theory realization: N coincident $D3$-branes probing $\widetilde{\mathbb{C}^2/\mathbb{Z}_2}$ with nonzero B-field.



$$X = \begin{pmatrix} & Q_{12} \\ Q_{21} & \end{pmatrix}, \quad Y = \begin{pmatrix} & \tilde{Q}_{12} \\ \tilde{Q}_{21} & \end{pmatrix}, \quad Z = \begin{pmatrix} \phi_1 & \\ & \phi_2 \end{pmatrix}$$

$$\mathscr{W}_{\hat{A}_1} = g_1 \mathrm{Tr}\left(\tilde{Q}_{21}\phi_1 Q_{12} - Q_{21}\phi_1\tilde{Q}_{12}\right) + g_2 \mathrm{Tr}\left(\tilde{Q}_{12}\phi_2 Q_{21} - Q_{12}\phi_2\tilde{Q}_{21}\right)$$

[Kachru, Silverstein 1998][Lawrence, Nekrasov, Vafa 1998]

Superconformal symmetry

- In the $\mathcal{N} = 4$ SYM theory, the superpotential $\mathscr{W} = g\mathrm{Tr}\left([X, Y]Z\right)$ is manifestly invariant under $\mathrm{SU}(3)_{XYZ} \subset \mathrm{SU}(4)_R$.
- Naively, in the $\mathcal{N} = 2$ SCFT of $\hat{A}_1$ quiver, the superpotential is invariant only under $\mathrm{SU}(2)_{XY} \times \mathrm{U}(1) \subset \mathrm{SU}(3)_{XYZ}$, while the generators transforming $X \leftrightarrow Z$ and $Y \leftrightarrow Z$ are broken.
- Accordingly, the superconformal symmetry is broken from $\mathrm{PSU}\left(2,2|4\right)$ to $\mathrm{SU}\left(2,2|2\right)$.

$SU(2)_{XZ}$ symmetry

- Focus on an $SU(2)_{XZ} \subset SU(3)_{XYZ}$ generated by

$$\left\{\mathfrak{e} = \mathscr{R}^2{}_3, \mathfrak{f} = \mathscr{R}^3{}_2, \mathfrak{h} = \mathscr{R}^2{}_2 - \mathscr{R}^3{}_3\right\}.$$

- Transformation rules are

$$\mathfrak{e}Q_{12} = \phi_1, \quad \mathfrak{e}Q_{21} = \phi_2, \quad \mathfrak{f}\phi_1 = Q_{12}, \quad \mathfrak{f}\phi_2 = Q_{21}, \quad \mathfrak{h}\phi_i = \phi_i, \quad \mathfrak{h}Q_{ij} = -Q_{ij},$$

and satisfy $[\mathfrak{e}, \mathfrak{f}] = \mathfrak{h}, [\mathfrak{h}, \mathfrak{e}] = 2\mathfrak{e}, [\mathfrak{h}, \mathfrak{f}] = -2\mathfrak{f}$.

- Naively, only $U(1)_{\mathfrak{h}} \subset SU(2)_{XZ}$ preserves the superpotential

$$\mathscr{W}_{\hat{A}_1} = g_1 \mathrm{Tr}\left(\tilde{Q}_{21}\phi_1 Q_{12} - Q_{21}\phi_1\tilde{Q}_{12}\right) + g_2 \mathrm{Tr}\left(\tilde{Q}_{12}\phi_2 Q_{21} - Q_{12}\phi_2\tilde{Q}_{21}\right).$$

Broken symmetries ⇒ quantum symmetries

Deform the universal enveloping algebra of $\mathfrak{su}(2)_{XZ}$ by modifying the actions on n-particle states.

Instead of the standard Leibniz rule (e.g. n=3)

$$\Delta(\mathfrak{j}) = \mathfrak{j} \otimes 1 \otimes 1 + 1 \otimes \mathfrak{j} \otimes 1 + 1 \otimes 1 \otimes \mathfrak{j},$$

the actions are given by the nontrivial co-product,

$$\Delta(\mathfrak{h}) = \mathfrak{h} \otimes 1 \otimes 1 + 1 \otimes \mathfrak{h} \otimes 1 + 1 \otimes 1 \otimes \mathfrak{h},$$

$$\Delta(\mathfrak{e}) = \mathfrak{e} \otimes \mathbb{K}_{\mathfrak{e}} \otimes \mathbb{K}_{\mathfrak{e}} + 1 \otimes \mathfrak{e} \otimes \mathbb{K}_{\mathfrak{e}} + 1 \otimes 1 \otimes \mathfrak{e},$$

$$\Delta(\mathfrak{f}) = \mathfrak{f} \otimes 1 \otimes 1 + \mathbb{K}_{\mathfrak{f}} \otimes \mathfrak{f} \otimes 1 + \mathbb{K}_{\mathfrak{f}} \otimes \mathbb{K}_{\mathfrak{f}} \otimes \mathfrak{f}.$$

Broken symmetries ⇒ quantum symmetries

- If we take

$$\mathbb{K}_{\mathfrak{e}}\left(Q_{12}, Q_{21}, \tilde{Q}_{12}, \tilde{Q}_{21}, \phi_1, \phi_2\right) = \left(\kappa Q_{21}, \kappa^{-1} Q_{12}, \kappa \tilde{Q}_{21}, \kappa^{-1} \tilde{Q}_{12}, \kappa^{-1} \phi_2, \kappa \phi_1\right),$$

$$\mathbb{K}_{\mathfrak{f}}\left(Q_{12}, Q_{21}, \tilde{Q}_{12}, \tilde{Q}_{21}, \phi_1, \phi_2\right) = \left(\kappa Q_{21}, \kappa^{-1} Q_{12}, \kappa \tilde{Q}_{21}, \kappa^{-1} \tilde{Q}_{12}, \kappa \phi_2, \kappa^{-1} \phi_1\right),$$

then $\Delta(\mathfrak{e})\mathscr{W}_{\hat{A}_1} = \Delta(\mathfrak{f})\mathscr{W}_{\hat{A}_1} = 0.$ $\kappa = g_2/g_1$

Therefore, a quantum enveloping algebra $U_\kappa[\mathfrak{su}(2)_{XZ}]$ preserves $\mathscr{W}_{\hat{A}_1}$.

- Similarly, the other broken generators can also be upgraded to quantum generators, and we can prove that the theory is invariant under a quantum enveloping algebra $U_\kappa[\mathfrak{psu}(2,2|4)]$, which is dual to a quantum group $\mathrm{PSU}\,(2,2|4)_\kappa$.

Conclusion and Outlook

- The manipulation can be generalized to all $\mathcal{N}$=2 SCFTs obtained from the $\mathcal{N} = 4$ SYM theory by orbifolds/orientifolds + exactly marginal deformations. The full $\mathcal{N} = 4$ superconformal symmetry is always preserved, albeit in a quantum deformed way.

- The symmetry enhancement from the manifest $\mathrm{SU}(2,2|2)$ to the hidden quantum group $\mathrm{PSU}(2,2|4)_\kappa$ helps us organize the spectrum of $\mathcal{N}$=2 SCFTs.

- It is interesting to further explore whether there exists a Yangian-like symmetry of the quantum group $\mathrm{PSU}(2,2|4)_\kappa$. [Andriolo, Bertle, Pomoni, XZ, Zoubos, in progress]

Thank you!

$\mathcal{N} = 4$ SYM

$$\Phi^{AB} = -\Phi^{BA} = \frac{1}{2}\epsilon^{ABCD}\bar{\Phi}_{CD} = \begin{pmatrix} 0 & Z & X & Y \\ -Z & 0 & \bar{Y} & -\bar{X} \\ -X & -\bar{Y} & 0 & \bar{Z} \\ -Y & \bar{X} & -\bar{Z} & 0 \end{pmatrix}$$

$$\mathcal{R}^A{}_B \Phi^{CD} = \delta^C_B \Phi^{AD} + \delta^D_B \Phi^{CA} - \frac{1}{2}\delta^A_B \Phi^{CD} .$$