Non-relativistic near-BPS corners of $\mathcal{N} = 4$ super Yang-Mills with SU$(1,1)$ symmetry

Stefano Baiguera
Niels Bohr Institute

Based on [S.B., T. Harmark, N. Wintergerst, arXiv:2009.03799]

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AdS/CFT correspondence

$\mathcal{N} = 4$ SYM with gauge group SU(N) $\leftrightarrow$ type IIB string theory on $\mathrm{AdS}_5 \times S^5$

Great success:

- Planar limit $N = \infty$ and the power of integrability [Minahan, Zarembo, 2002][Beisert, Kristjansen, Staudacher, 2003]

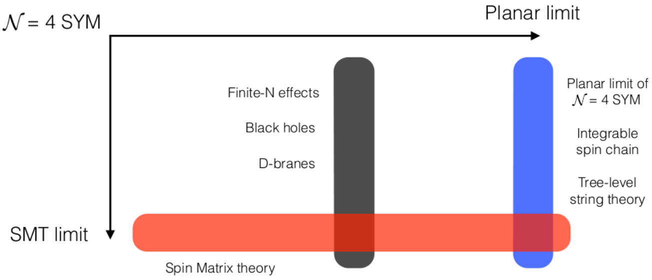
Problem:

- Gravity enters as $1/N$ perturbative corrections $\Rightarrow$ No access to black holes and D-branes

Spin Matrix Theory

Controlled finite N effects: Spin Matrix Theory limits [Harmark, Orselli, 2014].

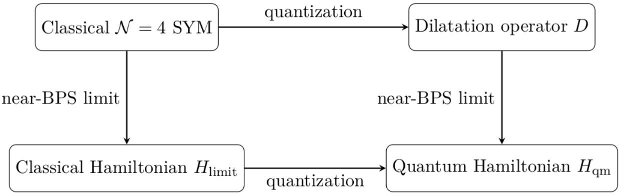
- Decoupling limits of $\mathcal{N} = 4$ SYM on $\mathbb{R} \times S^3 \Rightarrow$ the theory reduces to a subsector with only one-loop contributions of the dilatation operator [Harmark, Orselli, 2006][Harmark, Kristjansson, Orselli, 2006-07]
- Approach unitarity (near-BPS) bounds



Spin Matrix Theory is non-relativistic

- Emergent $\mathrm{U}(1)$ global symmetry corresponding to mass conservation
- The bulk duals are non-relativistic string theories with non-Lorentzian geometries [Harmark, Hartong, Obers, 2017][Harmark, Hartong, Menculini, Obers, Yan, 2018][Harmark, Hartong, Menculini, Obers, Oling, 2019]

Scheme of the procedure [Harmark, Wintergerst, 2019]:



Near-BPS limits with SU$(1,1)$ symmetries

Focus on BPS bounds

$$H \geq S_1 + \sum_{i=1}^{3} \omega_i Q_i \tag{1}$$

- S_1, S_2 Cartan generators for rotations on S^3
- Q_i Cartan generators of SU(4) R-symmetry group
- ω_i chemical potentials characterizing the bound

Spin Matrix Theory limit

$$g \to 0\,, \qquad \frac{H - S_1 - \sum_{i=1}^{3} \omega_i Q_i}{g^2 N} \text{ finite ,} \qquad N \text{ fixed} \tag{2}$$

For the purpose of this talk:

Sector	Combination of SU(4) Cartan charges $\sum_{i=1}^{3} \omega_i Q_i$
SU$(1,1\|1)$	$Q_1 + \frac{1}{2}(Q_2 + Q_3)$

Sphere reduction: general procedure

- Isolate the propagating modes in a given near-BPS limit from the quadratic classical Hamiltonian
- Integrate out additional non-dynamical modes giving rise to effective interactions in a given near-BPS limit
- Derive the interacting Hamiltonian from

$$H_{\text{int}} = \lim_{g \to 0} \frac{H - S_1 - \sum_{i=1}^{3} \omega_i Q_i}{g^2 N} \tag{3}$$

SU(1, 1|1) limit: classical interacting Hamiltonian

$$H_{\text{int}} = \frac{1}{2N}\left(\sum_{l=1}^{\infty}\frac{1}{l}\,\text{tr}\left(q_l^\dagger q_l\right) + \sum_{l=0}^{\infty}\text{tr}\left(F_l^\dagger F_l\right)\right) + \frac{1}{8N}\,\text{tr}\left(q_0^\dagger q_0\right) - \frac{1}{4N}\sum_{s=0}^{\infty}\frac{1}{s+1}\,\text{tr}\left([\Phi_s^\dagger,\Phi_s]q_0\right)\,. \tag{4}$$

where the blocks are

$$q_l \equiv \sum_{s\geq 0}\left([\Phi_s^\dagger,\Phi_{s+l}] + \frac{\sqrt{s+1}}{\sqrt{s+l+1}}\{\psi_s^\dagger,\psi_{s+l}\}\right)\,, \qquad F_l \equiv \sum_{s=0}^{\infty}\frac{[\psi_{s+l},\Phi_s^\dagger]}{\sqrt{s+l+1}}\,. \tag{5}$$

- Global $\mathrm{U}(1)$ symmetry: conservation of mass
- Commutes with all the generators of SU$(1,1|1)$ algebra
- The interaction is positive definite

SU$(1,1|1)$ limit: further results

Quantization prescription

We directly promote the Hamiltonian at quantum level

$\Downarrow$

Normal ordering gives rise to self-energy corrections

$\Downarrow$

The quantum interacting Hamiltonian matches with [Beisert, 2004][Bellucci, Casteill, Morales, 2005].

Semi-local formulation

Local interpretation of the fields with a positivity constraint on the modes

$$\Phi(t,x) = \sum_{n=0}^{\infty} \Phi_n(t) e^{i(n+\frac{1}{2})x}\,, \quad \psi(t,x) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n+1}} \psi_n(t) e^{i(n+1)x}\,. \tag{6}$$

$\Downarrow$

The interaction can be written in terms of fermionic superfields

$$\Psi(t,x,\theta,\theta^\dagger) = \psi + \theta\Phi + \frac{i}{2}\theta\theta^\dagger \partial_x \psi\,. \tag{7}$$

Future developments

- Investigation of SMT limits in sectors with $\mathrm{SU}(1,2)$ spin group (work in progress with T. Harmark, Y. Lei)
- Holographic investigations: TNC and SNC strings [Bergshoeff, Gomis, Gürsoy, Harmark, Hartong, Oling, Rosseel, Simsek, Yan, Zinnato, . . .]
- Relation to black holes for the $\mathrm{SU}(1,2|3)$ sector [Gutowski, Reall, 2004]

Thank you!