An Operator Product Expansion for Form Factors

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[2009.11297],[2012.xxxxx],[2012.yyyyy] with A. Sever and A. Tumanov
[2012.zzzzz] with L. Dixon and A. McLeod

VILLUM FONDEN

Motivation to study form factors

$\mathcal{A}(1,\dots,n)$
$= \langle 1,\dots,n|0\rangle$

$\mathcal{C}_{\mathcal{O}_1\dots\mathcal{O}_n}(x_1,\dots,x_n)$
$= \langle 0|\mathcal{O}_1(x_1)\dots\mathcal{O}_n(x_n)|0\rangle$

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$\mathcal{F}_{\mathcal{O}}(1,\dots,n;x) = \langle 1,\dots,n|\mathcal{O}(x)|0\rangle$

Bridge between on-shell and off-shell quantities:

$$\mathcal{F}_{\mathcal{O}}(1,\dots,n;q) = \int \frac{\mathrm{d}^4x}{(2\pi)^4}\,\mathrm{e}^{-iqx}\langle 1,\dots,n|\mathcal{O}(x)|0\rangle = \delta^4\left(q-\sum_{i=1}^{n}p_i\right)\langle 1,\dots,n|\mathcal{O}(0)|0\rangle$$

with $p_i^2 = 0$ but generically $q^2 \neq 0$

Motivation to study form factors

- Probe extent of methods and structures found for scattering amplitudes
  - Recursion relations Brandhuber, Spence, Travaglini, Yang (2010)], [Brandhuber, Gurdogan, Mooney, Travaglini, Yang (2011)], . . . , [Bolshov, Bork, Onishchenko (2018)], [Bianchi, Brandhuber, Panerai, Travaglini (2018)]
  - Color-kinematics duality [Boels, Kniehl, Tarasov, Yang (2012)], [Yang (2016)], [Lin, Yang (2020)]
  - On-shell diagrams, polytops and Graßmannians [Bork (2014)], [Frassek, Meidinger, Nandan, **MW** (2015)], [Bork, Onishchenko (2015-2017)]
  - Twistor space action [Koster, Mitev, Staudacher, **MW** (2016)], [Chicherin, Sokatchev (2016)]
  - Connected description [He, Zhang (2016)], [Brandhuber, Hughes, Rodolfo Panerai, Spence, Travaglini (2016)], [He, Liu (2016)]
  - . . .

Motivation to study form factors

- Probe extent of methods and structures found for scattering amplitudes
- Yield renormalisation group coefficients (dilatation operator, anomalous dimensions, beta functions) [**MW** (2014)], [Nandan, Sieg, **MW**, Yang (2014)], [Loebbert, Nandan, Sieg, **MW**, Yang (2015)], [Brandhuber, Kostacinska, Penante, Travaglini, Young (2016)], [Caron-Huot, **MW** (2016)], [Loebbert, Sieg, **MW**, Yang (2016)] also for SMEFT [Bern, Parra-Martinez, Sawyer (2019-2020)],. . .

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- Sudakov form factors reveal IR structure of amplitudes
  $\rightarrow$ non-planar cusp anomalous dimension [Boels, Kniehl, Tarasov, Yang (2012)], [Boels, Kniehl, Yang (2015)], [Huber, von Manteuffel, Panzer, Schabinger, Yang (2019)]

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- Form factor of $\mathcal{L}$ describes $H \rightarrow \textit{gluons}$ amplitude in $m_{\text{top}} \rightarrow \infty$ limit
  Three-point two-loop remainder in $\mathcal{N} = 4$ SYM theory agrees with the maximally transcendental part of its counterpart in QCD [Brandhuber, Travaglini, Yang (2012)] (Similar for next order in $m_{\text{top}} \rightarrow \infty$ limit [Brandhuber, Kostacinska, Penante, Travaglini (2017-2018)], [Jin, Yang (2018)])

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- Integrability at strong coupling via Y-system [Maldacena, Zhiboedov (2010)], [Gao, Yang (2013)] and weak coupling [Frassek, Meidinger, Nandan, **MW** (2015)]

This talk

Today:

- Integrability for form factors (in expansion around near-colinear limit) at finite coupling [Sever, Tumanov, **MW** (2020 & to appear)]
- Perturbative bootstrap of form factors for generic kinematics at three-, four- and five-loop order [Dixon, McLeod, **MW** (to appear)]

Focus: Planar form factor of chiral half of stress tensor supermultiplet ($\frac{1}{2}$BPS, $\ni \mathcal{L}$, $\operatorname{tr} Z^2$) in $\mathcal{N} = 4$ SYM theory

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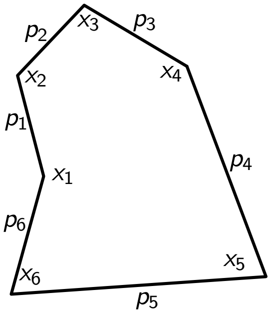
Dual description via Wilson loops

Dual coordinates x_i: $x_{i+1} - x_i = p_i$

Amplitudes: $\sum_{i=1}^{n} p_i = 0 \Rightarrow$ closed null polygonal Wilson loop



[Alday, Maldacena (2007)], [Drummond, Korchemsky, Sokatchev (2007)], [Brandhuber, Heslop, Travaglini (2007)], [Drummond, Henn, Korchemsky, Sokatchev (2007)], ...

Dual description via Wilson loops

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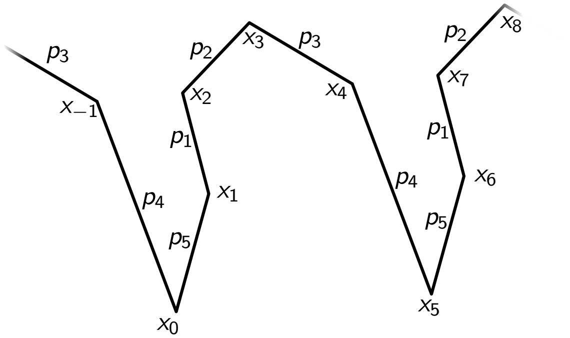
Form factors: $\sum_{i=1}^{n} p_i = q \Rightarrow$ periodic null polygonal Wilson loop



$$x_{i+n} = x_i + q$$

[Alday, Maldacena (2007)], [Maldacena, Zhiboedov (2010)], [Brandhuber, Spence, Travaglini, Yang (2010)], [Ben-Israel, Sever, Tumanov (2018)], [Bianchi, Brandhuber, Panerai, Travaglini (2018)]

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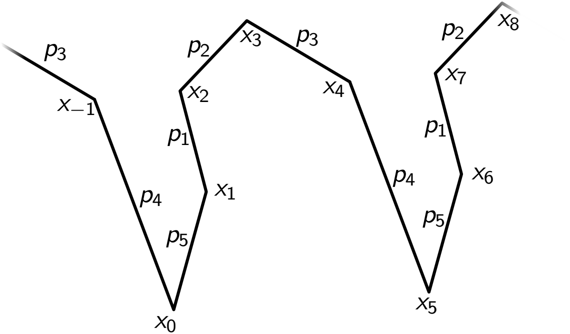

Manifest dual conformal symmetry: $x_{i+n} = x_i + q \rightarrow P(x_i)$

[Alday, Maldacena (2007)], [Maldacena, Zhiboedov (2010)], [Brandhuber, Spence, Travaglini, Yang (2010)], [Ben-Israel, Sever, Tumanov (2018)], [Bianchi, Brandhuber, Panerai, Travaglini (2018)]

Idea of the Form Factor Operator Product Expansion

Amplitudes: Pentagon Operator Product Expansion

→ Closed Wilson loops (in near-colinear limit) at any 't Hooft coupling

[Basso, Sever, Viera (2013-2015)], [Basso, Caetano, Cordova, Sever, Vieira (2014-2015)], [Belitsky (2014-2016)], ...

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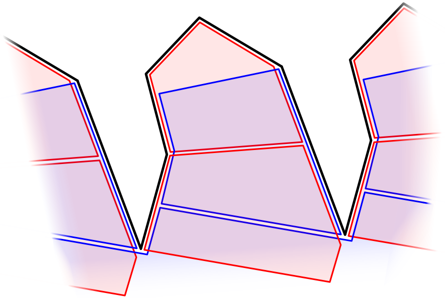
Form factors: Operator Product Expansion based on periodic Wilson loos! [Sever, Tumanov, **MW** (2020)]

Idea of the Form Factor Operator Product Expansion

Decompose n-sided periodic Wilson loop into $n-2$ pentagons and a 2-sided periodic Wilson loop



[Sever, Tumanov, **MW** (2020)]

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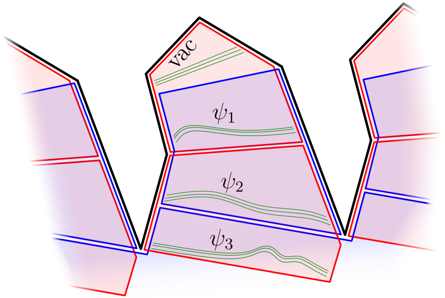

Idea behind Form Factor Operator Product Expansion:

$$\text{vacuum} \xrightarrow{\mathcal{P}} \psi_1 \xrightarrow{\mathcal{P}} \psi_2 \xrightarrow{\mathcal{P}} \dots \xrightarrow{\mathcal{P}} \psi_{n-2} \dashv \mathcal{F}$$

$\mathcal{P}$ = pentagon transition
$\mathcal{F}$ = form factor transition

[Sever, Tumanov, **MW** (2020)]

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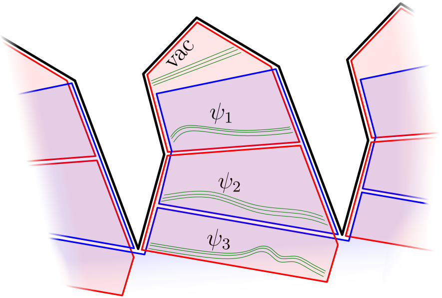
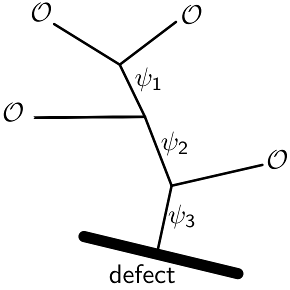


$\leftrightarrow$



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$\mathcal{P}$ = pentagon transition ∼ three-point function
$\mathcal{F}$ = form factor transition ∼ one-point function

[Sever, Tumanov, **MW** (2020)]

Regularization

UV divergences at cusps $\Rightarrow$ Consider UV finite ratio

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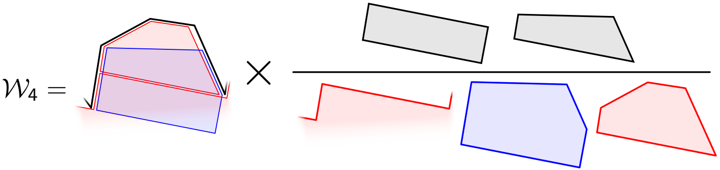
Regularization

UV divergences at cusps ⇒ Consider UV finite ratio

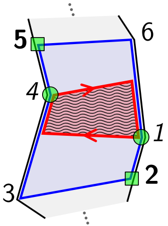


$$\mathcal{W}_n = \frac{\langle W_{n\text{-pt ff}}\rangle \times \langle W_{\text{2nd square}}\rangle \langle W_{\text{3rd square}}\rangle \cdots}{\langle W_{\text{2-pt ff}}\rangle \times \langle W_{\text{1st pentagon}}\rangle \langle W_{\text{2nd pentagon}}\rangle \cdots}$$

[Sever, Tumanov, **MW** (2020)]

Each square has τ_i, σ_i, ϕ_i conjugate to GKP twist, conformal spin and angular momentum in the transverse plane

Upper $(n-3)$ middle squares parametrized as for amplitudes:



$$u_i = u_{i+3} \equiv \frac{x^2_{i-1,i+1}x^2_{i-2,i+2}}{x^2_{i-1,i+2}x^2_{i+1,i-2}}$$

$$\frac{1}{u_2} = 1 + e^{2\tau}$$
$$\frac{1}{u_3} = 1 + (e^{-\tau} + e^{\sigma+i\phi})(e^{-\tau} + e^{\sigma-i\phi})$$
$$\frac{u_1}{u_2 u_3} = e^{2\sigma+2\tau}$$

[Basso, Sever, Viera (2013)]

Parametrization

Each square has τ_i, σ_i, ϕ_i conjugate to GKP twist, conformal spin and angular momentum in the transverse plane

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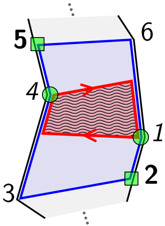

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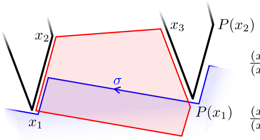
[Basso, Sever, Viera (2013)]

Lowest square parametrized as



$$\frac{(x_2-P(x_2))^2(P(x_1)-P(P(x_1)))^2}{(x_2-P(x_1))^2(P(x_2)-P(P(x_1)))^2} = \left(1+e^{2\tau}\right)^2$$

$$\frac{(x_1-P(x_1))^2(x_3-P(x_3))^2}{(x_1-x_3)^2(P(x_1)-P(x_3))^2} = \left(1+e^{-2\tau}+e^{2\sigma}\right)^2$$

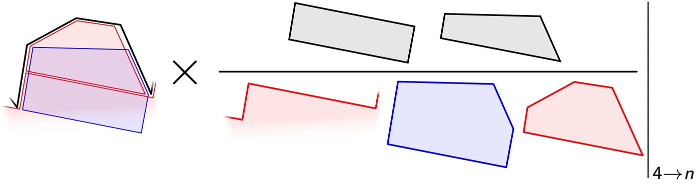
and $\phi = 0$

[Sever, Tumanov, **MW** (2020)]

Statement of the Form Factor Operator Product Expansion

Main statement:

$\mathcal{W}_n =$



$$= \sum_{\psi_1,\dots,\psi_{n-2}} e^{\sum_j(-E_j\tau_j+ip_j\sigma_j+im_j\phi_j)}\mathcal{P}(0|\psi_1)\dots\mathcal{P}(\psi_{n-3}|\psi_{n-2})\,\mathcal{F}(\psi_{n-2})$$

- GKP energy E_i, momentum p_i and angular momentum m_i known at any coupling via integrability of GKP flux tube [Basso (2010)]
- Pentagon transition $\mathcal{P}$ known at any coupling [Basso, Sever, Viera (2013-2015)], [Basso, Caetano, Cordova, Sever, Vieira (2014-2015)], [Belitsky (2014-2016)]
- New universal building block: Form factor transition $\mathcal{F}$

[Sever, Tumanov, **MW** (2020)]

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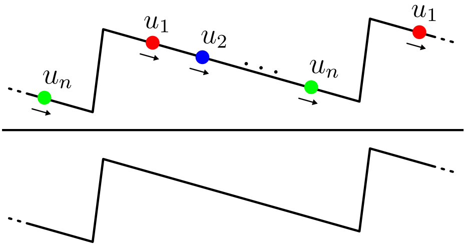
Form factor transition

GKP eigenstates parametrized by

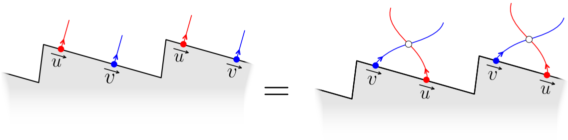
- the number of excitations N
- their species $\mathbf{a} = \{a_1, \ldots, a_N\}$
- their Bethe rapidities $\mathbf{u} = \{u_1, \ldots, u_N\}$

Form factor transition

$F_{\mathbf{a}}(\mathbf{u}) \equiv$ 

Satisfies axioms $\Rightarrow$ Bootstrap at finite coupling

Axiom I: Watson



$$F_{\ldots,a_j,a_{j+1}}(\ldots,u_j,u_{j+1},\ldots) = S(u_j,u_{j+1})_{a_j a_{j+1}}^{b_j b_{j+1}} F_{\ldots,b_j,b_{j+1},\ldots}(\ldots,u_{j+1},u_j,\ldots)$$

[Sever, Tumanov, **MW** (2020)]

Axiom II: Singlet

Two-sided periodic Wilson loop is invariant under:

- $U(1)_\phi$: rotations in the two-dimensional transverse plane
- $SU(4)_R$: R-symmetry group

[Sever, Tumanov, **MW** (2020)]

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$\Rightarrow$ Form factor transition is $U(1)_\phi \times SU(4)_R$ singlet

$$\forall \mathcal{M} \in U(1)_\phi \times SU(4)_R : \qquad F_{a_1,\dots,a_n}(\mathbf{u}) = \mathcal{M}_{a_1}^{b_1} \dots \mathcal{M}_{a_n}^{b_n} F_{b_1,\dots,b_n}(\mathbf{u})$$

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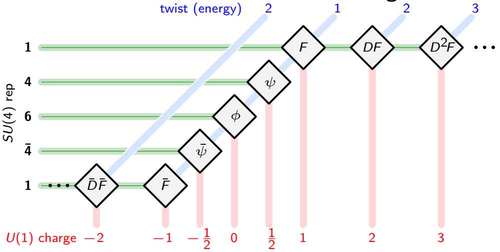
Fundamental GKP excitations all charged under $U(1)_\phi \times SU(4)_R$



picture:
[Basso, Sever, Viera (2014)]

[Sever, Tumanov, **MW** (2020)]

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$\Rightarrow$ FF transition cannot absorb a fundamental single-particle state

$\Rightarrow$ Only singlet states with even Born-level energy can contribute to the FF transition

$\Rightarrow$ At any loop order only even powers of $e^{-\tau}$

[Sever, Tumanov, **MW** (2020)]

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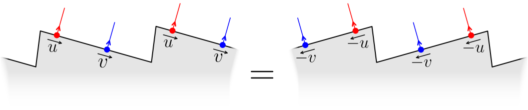
[Sever, Tumanov, **MW** (2020)]

Axiom III: Reflection

Discrete $\mathbb{Z}_2$ symmetry: flipping the direction of the two edges ($\sigma \to -\sigma$)



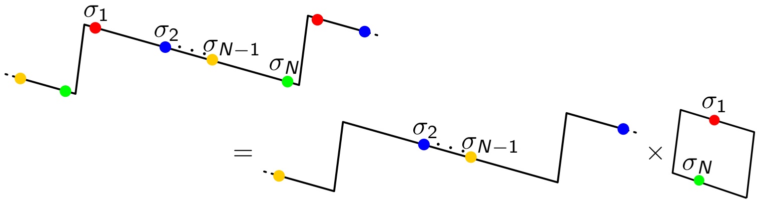
$$F_{a_1,\ldots,a_N}(u_1,\ldots,u_N) = F_{a_N,\ldots,a_1}(-u_N,\ldots,-u_1)$$

[Sever, Tumanov, **MW** (2020)]

Axioms IV: Square limit

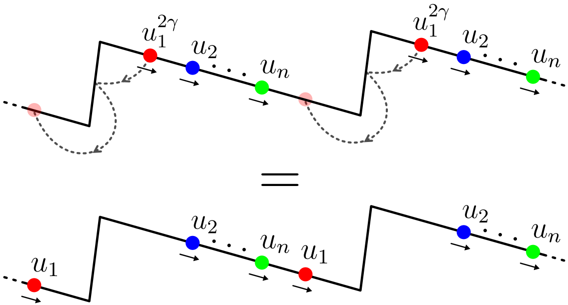
$$\lim_{u_1 \to u_n} F_{\mathbf{a}}(\mathbf{u}) = \frac{-i\delta_{a_n,\bar{a}_1}}{\mu_{a_1}(u_1)} \frac{F_{a_2,\ldots,a_{n-1}}(u_2,\ldots,u_{n-1})}{u_1 - u_n - i\epsilon}$$
$$\pm \left(S(u_1,u_n) \prod_{1<j<n} S(u_1,u_j) S(u_j,u_n) \right)_{\mathbf{a}}^{\mathbf{b}}$$
$$\times \frac{-i\delta_{b_n,\bar{b}_1}}{\mu_{b_1}(u_1)} \frac{F_{b_2,\ldots,b_{n-1}}(u_2,\ldots,u_{n-1})}{u_n - u_1 - i\epsilon}$$

with $+/-$ for bosons/fermions



[Sever, Tumanov, **MW** (2020)]

Axiom V: Crossing



$$F(u_1^{2\gamma}, u_2, \ldots, u_n) = F(u_2, \ldots, u_n, u_1)$$

Mirror transformation γ: $p(u^\gamma) = iE(u)$ and $E(u^\gamma) = ip(u)$

[Sever, Tumanov, **MW** (2020)]

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Leading weak-coupling contributions

Singlet axiom

- ⇒ Leading weak-coupling contribution in OPE limit $\tau \to \infty$ from singlet states with $E_{g=0} = 2$
- ⇒ Two-particles $\to$ σ couples to $p(u_1) + p(u_2)$
- ⇒ Hard!

Leading weak-coupling contributions

Singlet axiom

- $\Rightarrow$ Leading weak-coupling contribution in OPE limit $\tau \to \infty$ from singlet states with $E_{g=0} = 2$
- $\Rightarrow$ Two-particles $\to$ σ couples to $p(u_1) + p(u_2)$
- $\Rightarrow$ Hard!

Three $E_{g=0} = 2$ singlet states built from two fundamental excitations:

$$\begin{aligned} |u_1, u_2; \sigma_1, \sigma_2\rangle^{\xi} = \psi^{\xi}_{F\bar{F}}(u_1, u_2; \sigma_1, \sigma_2)|F(\sigma_1)\bar{F}(\sigma_2)\rangle \\ + \psi^{\xi}_{\psi\bar{\psi}}(u_1, u_2; \sigma_1, \sigma_2)|\psi(\sigma_1)\bar{\psi}(\sigma_2)\rangle \\ + \psi^{\xi}_{\phi\bar{\phi}}(u_1, u_2; \sigma_1, \sigma_2)|\phi(\sigma_1)\bar{\phi}(\sigma_2)\rangle \end{aligned}$$

Labeled by leading contribution for $\sigma_1 \ll \sigma_2$: $\xi = F\bar{F}, \psi\bar{\psi}, \phi\bar{\phi}$

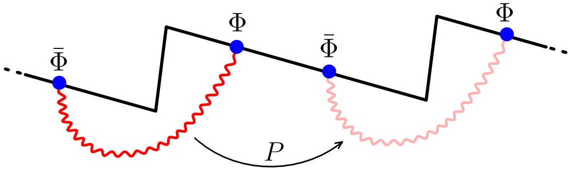
Wave functions $\psi^{\xi}_{\Phi\bar{\Phi}}(u_1, u_2; \sigma_1, \sigma_2)$ [Sever, Tumanov, **MW** (to appear)]

(Effective one-particle singlet excitations do not contribute)

Born-level form factor transition

Propagator:

$$\langle F|\Phi(\sigma_1)\bar{\Phi}(\sigma_2)\rangle = \frac{1}{(e^{\sigma_1+\sigma_2} + e^{-\sigma_1-\sigma_2} + 2e^{\sigma_1-\sigma_2})^{2s}}$$

$=$ 

with $s = \frac{1}{2}, 1, \frac{3}{2}$ for $\Phi = \phi, \psi, F$

[Sever, Tumanov, **MW** (2020)]

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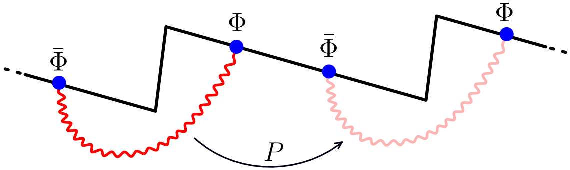
$=$ 

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$$F_\xi(u_1,u_2)=\sum_{\Phi=\phi,\psi,F}\int_{-\infty<\sigma_1\leq\sigma_2<+\infty}\langle F|\Phi(\sigma_1)\bar{\Phi}(\sigma_2)\rangle\psi^{\xi}_{\Phi\bar{\Phi}}(u_1,u_2;\sigma_1,\sigma_2)$$

[Sever, Tumanov, **MW** (2020)]

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Final result at Born level:

$$F_{\phi^i \phi^j}(u,v) = -\,\delta^{ij} \times \frac{4}{g^2\,(u-v-2i)\,(u-v-i)} \frac{\Gamma\,(iu-iv)}{\Gamma\left(\frac{1}{2}+iu\right)\Gamma\left(\frac{1}{2}-iv\right)}$$

$$F_{\psi^A \bar{\psi}^B}(u,v) = +\,\delta^{AB} \times \frac{2}{g^2}\, u \sinh(\pi u)\, \delta(u-v)$$

$$F_{F\bar{F}}(u,v) = -\,1 \times \frac{2}{g^2}\left(u^2+\tfrac{1}{4}\right)\cosh(\pi u)\,\delta(u-v)$$

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Minimal solution consistent with square limit axiom

[Sever, Tumanov, **MW** (2020)]

Matching data

One-loop ratio (extracted from [Brandhuber, Spence, Travaglini, Yang (2010)]):

$$\mathcal{W}_3^{\text{1-loop}} = 4\sigma^2 - 2\,\text{Li}_2(-e^{-2\tau}) + 2\,\text{Li}_2(-e^{-2\tau} - e^{2\sigma}) + 2\,\text{Li}_2(-e^{-2\tau} - e^{-2\sigma}(1+e^{-2\tau})^2) + \frac{\pi^2}{3}$$

Expansion:

$$\mathcal{W}_3^{\text{1-loop}} = 2\,e^{-2\tau}\left(1 - 2\,\sigma\,e^{-2\sigma} - 4\cosh^2(\sigma)\log\left(1+e^{-2\sigma}\right)\right) + O(e^{-4\tau})$$

$\Rightarrow$ Perfect match with FFOPE!

Two-loop check

Loop corrections to E [Basso (2010)] are only source of polynomial terms in τ

$$\mathcal{W}_3 = \sum_{\psi} e^{-E_\psi \tau + i p_\psi \sigma} \mathcal{P}(0|\psi) \mathcal{F}(\psi)$$

$\Rightarrow$ Born-level $\mathcal{F}$ provides $\tau^{\ell-1}$ terms of ℓ-loop $\mathcal{W}_3$!

[Sever, Tumanov, **MW** (2020)]

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Translate between ratio $\mathcal{W}$ and the remainder $\mathcal{R}$:

$$\mathcal{W}_n = \exp\left[\frac{\Gamma_{\text{cusp}}}{4} \mathcal{W}_n^{\text{1-loop}}\right] \times \exp(\mathcal{R}_n) \ ,$$

$\Gamma_{\text{cusp}} = 4g^2 + \dots$ cusp anomalous dimension [Beisert, Eden, Staudacher (2006)]

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$\Rightarrow$ Perfect match with two-loop remainder [Brandhuber, Travaglini, Yang (2012)] at order $\tau e^{-2\tau}$!

[Sever, Tumanov, **MW** (2020)]

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Simplest solution to axioms for fermions and gluons:

$$F_{\psi^A \bar{\psi}^B}(u,v) = \delta^{AB} \frac{2\pi}{\mu_\psi(u)} \delta(u-v)$$

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[Sever, Tumanov, **MW** (to appear)]

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Bootstrapping form factor transitions at finite coupling

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Solution to axioms for scalars not unique

$\Rightarrow$ Parametrize freedom and fix via data!

[Sever, Tumanov, **MW** (to appear)]

Form factor bootstrap

No data beyond two-loop $\Rightarrow$ Make your own!

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Perturbative bootstrap for 6-point amplitudes up to 7 loops and 7-point amplitudes up to 4 loops [Caron-Huot, Dixon, Drummond, Duhr, Dulat, Foster, Gurdogan, Harrington, Henn, von Hippel, McLeod, Papathanasiou, Pennington, Spradlin (2011-2020)]

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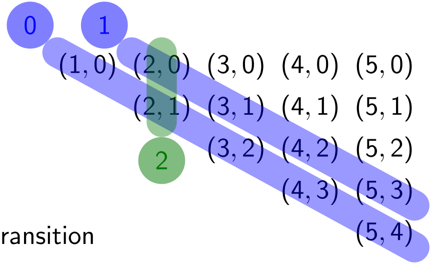

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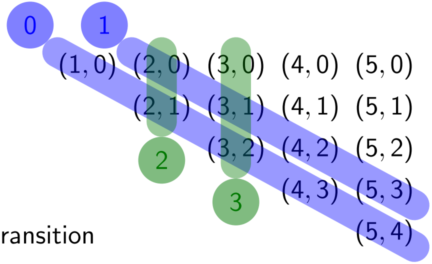

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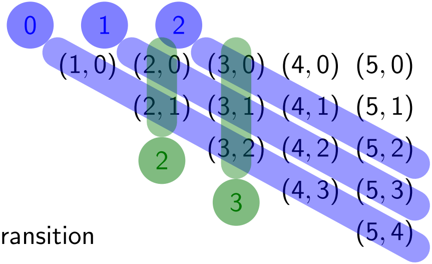

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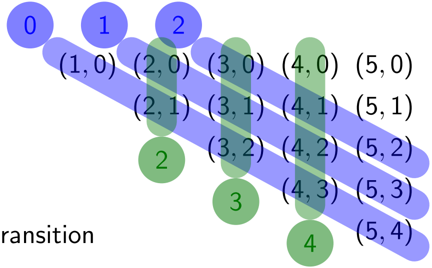

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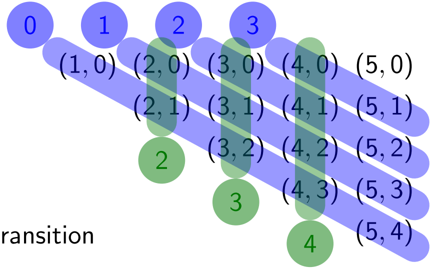

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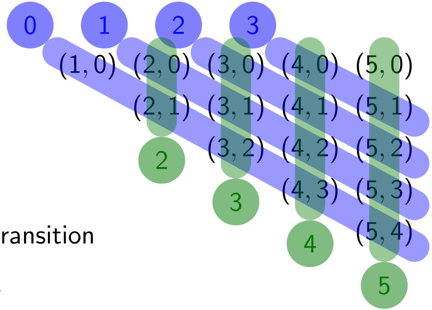

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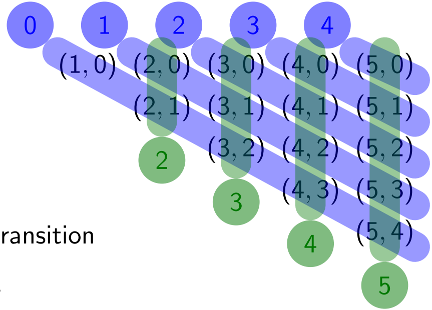

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$\Rightarrow$ Match up to five-loop order and with minimal solution to the axioms at strong coupling

Factorized ansatz

Higher orders in $e^{-2\tau}$ $\Rightarrow$ $2n$-particle singlet states contribute

Factorized ansatz: e.g.

$$F_{F\psi\bar{\psi}\bar{F}}(u_1, u_2, u_3, u_4) = F_{F\bar{F}}(u_1, u_4) F_{\psi\bar{\psi}}(u_2, u_3)$$

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Conclusions and outlook

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- Operator product expansion for form factors
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  $\Rightarrow$ Form factors in near-collinear expansion at any coupling

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Thank you!