Baxter Equation for Boundary Integrability

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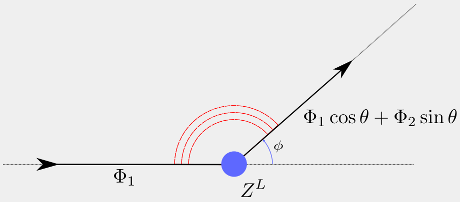
Based on
JHEP 07 (2020) 042 with David Grabner and Nikolay Gromov
arXiv:2012.xxxxxx with Nikolay Gromov and Nicolò Primi
arXiv:2012.xxxxxx

King's College London

November 26, 2020

Setup

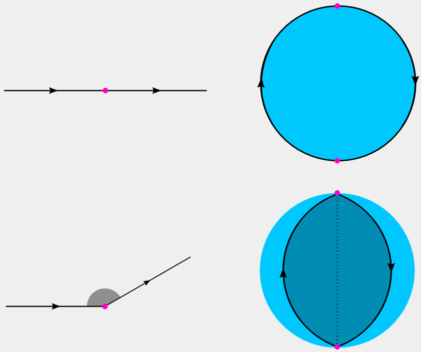


$$W = \frac{1}{N} \operatorname{tr} \mathrm{P} \exp \int_0^\infty dt\, 4\pi g \left(i A \cdot x'(t) + \Phi_1 |x'(t)| \right) \times Z(0)^J \times$$
$$\mathrm{P} \exp \int_0^\infty ds\, 4\pi g \left(i A \cdot x'(s) + (\Phi_1 \cos\theta + \Phi_2 \sin\theta) |x'(s)| \right)$$

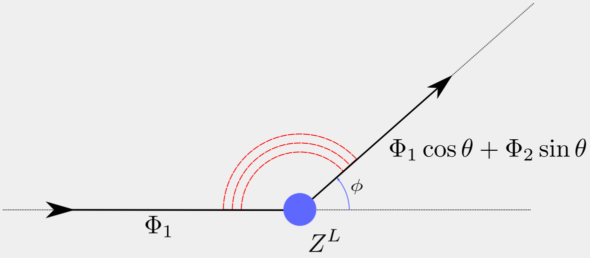
$$Z = \frac{1}{\sqrt{2}} (\Phi_3 + i \Phi_4)$$

$$\langle W \rangle = \left(\frac{\Lambda}{\epsilon} \right)^{\Delta}$$

The Cusp as a Local Operator



- Wilson loop with circular n-gon contour $\to$ n-point function of local operators (Drukker and Forini 2011; Cavaglia, Gromov, and Levkovich-Maslyuk 2018; Dorn 2020)
- Insert local operators at these cusps (Cavaglia, Gromov, and Levkovich-Maslyuk 2018; McGovern 2019)

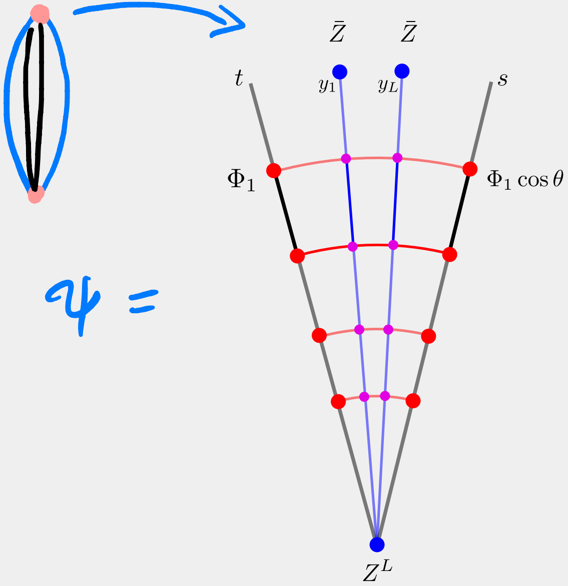


$g \to 0$, $\theta \to i\infty$ with $\hat{g}^2 \equiv g^2 \exp(-i\theta)$ kept fixed (Erickson, Semenoff, and Zarembo 2000; Correa, Henn, Maldacena and Sever 2012)

- Gluons and fermions decouple
- Only the contributions with the highest power of $\cos\theta$ will survive:
  - For $L = 0$, only “ladder” diagrams contribute
  - For $L > 0$, only “fishnet” diagrams contribute
- Φ_2 drops out

The Open Fishchain

CFT Wavefunction



$\Psi =$

Fishchain Hamiltonian

Graph-destroying Operator

$$\partial_t \partial_s \prod_{j=1}^{L} \Box_{y_j} \sum_{\alpha=0}^{l} \psi_\alpha = (4\hat{g}^2)^{L+1} \frac{|y_0'||y_{L+1}'|}{\prod_{i=0}^{L}(y_i - y_{i+1})^2} \sum_{\alpha=0}^{l-1} \psi_\alpha$$

4D Hamiltonian

$$\hat{H} = \frac{1}{(4\hat{g}^2)^{L+1}} \frac{\prod_{i=0}^{L}(y_i - y_{i+1})^2}{|y_0'||y_{L+1}'|} \partial_t \partial_s \prod_{j=1}^{L} \Box_{y_j} - 1$$

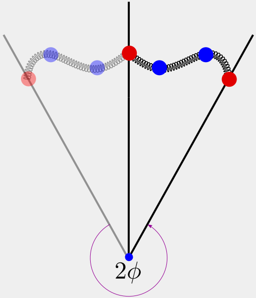
Embed in 6D

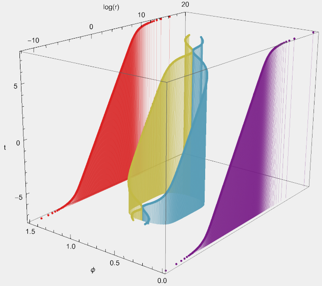
$$L = \frac{|\dot{X}_0||\dot{X}_{L+1}|}{2} + \sum_{i=1}^{L} \left(\frac{\dot{X}_i^2}{2} \right) + \prod_{k=0}^{L} (X_k \cdot X_{k+1})^{-\frac{1}{L+1}}$$

Similar construction to (Gromov and Sever 2019)

Equations of Motion

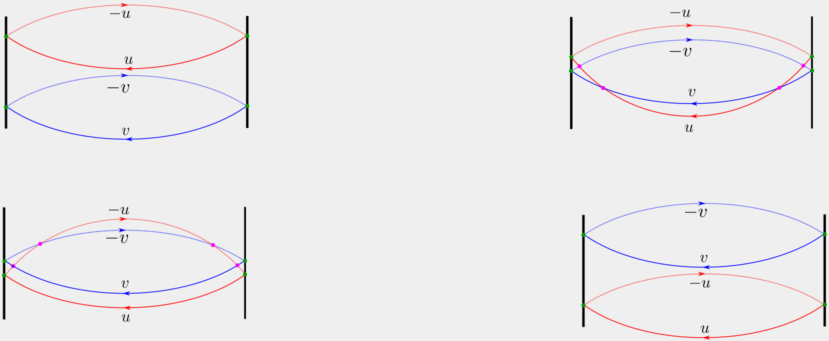
- We have a multi-particle classically integrable system
- Bulk particles y_i have four degrees of freedom
- Boundary particles constrained to move on ~~light-rays~~ fixed rays in 6d — implemented by introducing mirror particles





$$\hat{\mathbb{T}}^{4}(u) = \mathrm{tr}\hat{\bar{\mathbb{L}}}_{L}^{4}(-u)\cdot\hat{\bar{\mathbb{L}}}_{L-1}^{4}(-u)\dots\hat{\bar{\mathbb{L}}}_{1}^{4}(-u)\cdot\hat{\mathbb{K}}^{4}(u)$$

$$\cdot\,\hat{\mathbb{L}}_{1}^{4}(u)\cdot\hat{\mathbb{L}}_{2}^{4}(u)\dots\hat{\mathbb{L}}_{L}^{4}(u)\cdot G^{4}\cdot\hat{\bar{\mathbb{K}}}^{4}(u)\cdot G^{4\,\mathsf{T}}$$



$$[\hat{\mathbb{T}}^{4}(u),\hat{\mathbb{T}}^{4}(v)] = 0$$

General L

$$
\begin{aligned}
\mathbb{T}^{\mathbf{1}}(v) &= 1 \\
\mathbb{T}^{\mathbf{4}}(v) &\equiv \frac{P^{\mathbf{4}}_{L+1}(v^2)}{\hat{g}^{2L+2}} \\
\mathbb{T}^{\mathbf{6}}(v) &\equiv A(2v)\frac{v^2+1}{v^2}\frac{P^{\mathbf{6}}_{2L+2}(v^2)}{\hat{g}^{4L+4}} \\
\mathbb{T}^{\bar{\mathbf{4}}}(v) &= A(2v)A(2v+i)A(2v-i)\frac{(v^2+\frac{9}{4})(v^2+\frac{1}{4})^{2L+1}P^{\bar{\mathbf{4}}}_{L+1}(v^2)}{\hat{g}^{6L+6}} \\
\mathbb{T}^{\bar{\mathbf{1}}}(v) &= A^2(2v)A(2v+i)A(2v-i)A(2v+2i)A(2v-2i)\frac{(v^2+4)(v^2+1)^{2L+2}v^{4L+2}}{\hat{g}^{8L+8}}
\end{aligned}
$$

- P^{λ}_{L+1} are polynomials which contain other non-trivial integrals of motion
- $4L+2$ conserved charges in total, which equates the number of degrees of freedom of the problem, guaranteeing integrability

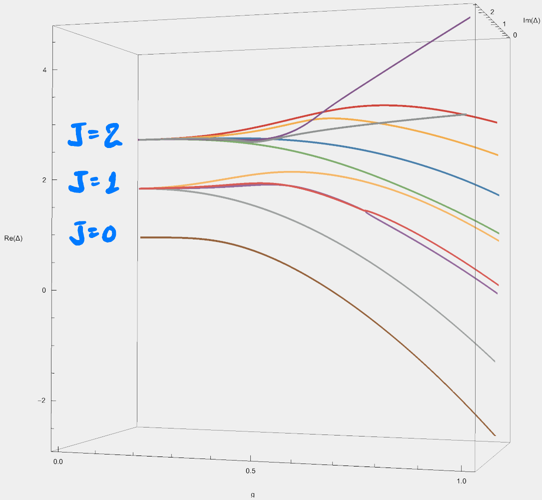
$$\mathbb{T}^{\mathbf{1}}(v+i)Q(v+2i)+\mathbb{T}^{\mathbf{4}}(v+i/2)Q(v+i)$$
$$+\mathbb{T}^{\mathbf{6}}(v)Q(v)+\mathbb{T}^{\bar{\mathbf{4}}}(v-i/2)Q(v-i)+\mathbb{T}^{\bar{\mathbf{1}}}(v-i)Q(v-2i)=0\,.$$

After identifying $Q(v)=\frac{e^{\pi L v}q(v)\Gamma(-iv)\hat{g}^{2i(L+1)v}\Gamma(iv+1)^{-2L-1}}{\Gamma\left(-iv-\frac{1}{2}\right)\Gamma(iv+2)}$ we get

$$\frac{P^{\mathbf{6}}_{2L+2}(v^2)}{v^{2L+3}}q(v)=$$
$$-(v+i)^{2L+1}q(v+2i)+\frac{v+\frac{i}{2}}{v(v+i)}P^{\mathbf{4}}_{L+1}\left((v+\tfrac{i}{2})^2\right)q(v+i)$$
$$-(v-i)^{2L+1}q(v-2i)+\frac{v-\frac{i}{2}}{v(v-i)}P^{\bar{\mathbf{4}}}_{L+1}\left((v-\tfrac{i}{2})^2\right)q(v-i)$$

Results

L = 1, $\varphi = 0$



Excited States

- For $L = 1$, we should get only one state
- Infinitely many excited states
- Have same R-charge as $L = 1$ case, so can't be more othogonal insertions
- Realised by (Cavaglia, Gromov, and Levkovich-Maslyuk 2018) that these correspond to "parallel" insertions
- Can take the straight-line limit of the cusp and obtain the 1d defect CFT that lives on the 1/2-BPS Maldacena-Wilson line

Defect CFT

Operators in the defect CFT

$$\mathcal{W} = \mathrm{tr}\,\mathrm{P}\exp\int_{-\infty}^{+\infty} dt(iA\cdot\dot{x} + \Phi_{||}|\dot{x}|) \tag{1}$$

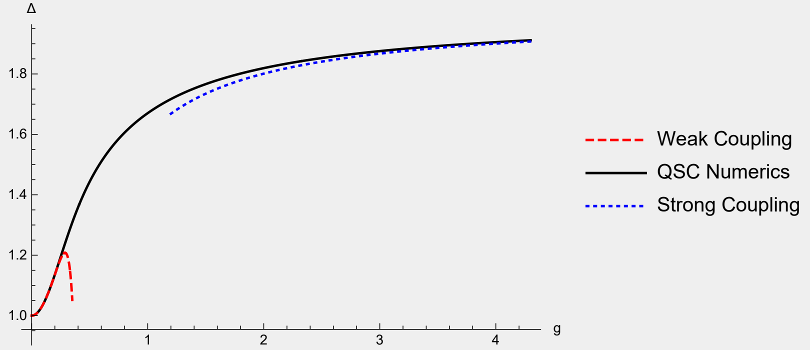
- $SO(5)_R$ out of the $SO(6)_R$
- $SO(2,1)\times SO(3)$ of $SO(2,4)$
- Together with fermionic symmetries, it preserves $OSp(4^*|4)$
- We are interested in operators that transform under the singlet of the $SO(5)_R$
- These correspond to excited states (labelled by J) with $L=0$
- Can only do second-principles derivation of Baxter equation from QSC

Results

J = 1

- At weak coupling, we have only one singlet — $\Phi_{||}$
- At strong coupling, the lowest dimension singlet — $\delta_{a\,b} y^a y^b$

Indeed the QSC interpolates between these two states



Weak Coupling

Our **five-loop** result for $\Delta \simeq 1 + g^2\gamma_1 + \cdots + g^{10}\gamma_5$

$$
\begin{aligned}
\gamma_1 &= 4 \\
\gamma_2 &= -16 \\
\gamma_3 &= -\frac{56\pi^4}{45} + 128\ , \\
\gamma_4 &= \frac{272}{135}\pi^6 + \frac{128}{3}\pi^2 - \frac{64}{3}\pi^2\zeta_3 + 128\zeta_3 - 160\zeta_5 - 1280\ , \\
\gamma_5 &= -\frac{7328}{2835}\pi^8 - \frac{64}{2835}\pi^6 - \frac{896}{45}\pi^4 - \frac{2560}{3}\pi^2 + \frac{64}{3}\pi^4\zeta_3 + \frac{512}{3}\pi^2\zeta_3 \\
&\quad + \frac{448}{3}\pi^2\zeta_5 - 384(\zeta_3)^2 - 1024\zeta_3 - 640\zeta_5 + 2688\zeta_7 + 14336
\end{aligned}
$$

- Up to two-loops by (Alday and Maldacena 2007; Brüser, Caron-Huot, and Henn 2018)

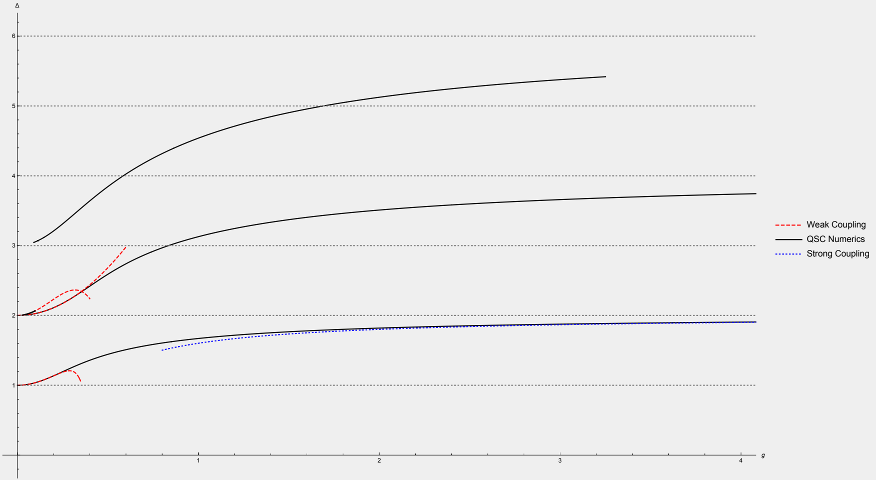
Strong Coupling

Our **four-order** result at strong coupling

$$\Delta = 2 - \frac{5}{\sqrt{\lambda}} + \frac{295}{24}\frac{1}{\lambda} - \frac{305}{16}\frac{1}{\lambda^{3/2}} + \mathcal{O}\left(\frac{1}{\lambda^2}\right) \ .$$

- First two orders by (Giombi, Roiban, and Tseytlin 2017)
- Third order (cf. Meneghelli LIJC 2020)

General J



Summary

- We have shown that the Feynman Diagrams of the CFT wavefunction in the classical limit are correctly resummed by a discretised open string — the open fishchain
- It is a spin-chain of particles which is integrable both classically and at the quantum level
- Derived AdS/CFT for these special observables in $\mathcal{N} = 4$ SYM

$$\text{CFT Wavefunction} \longleftrightarrow \text{Bethe Wavefunction}$$

$$\text{Q-functions} \longleftrightarrow \text{Baxter Polynomials}$$

$$\langle v_1, v_2, \ldots, v_L | \psi \rangle = q(v_1)q(v_2)\cdots q(v_L)$$

Thank You :)